Question Booklet

2015 Examination for Japanese University Admission for International Students

Mathematics (80 min.)

【Course 1 (Basic), Course 2 (Advanced)】

※ **Choose one of these courses and answer its questions only.**

I Rules of Examination

1. Do not leave the room without proctor's permission.
2. Do not take this question booklet out of the room.

II Instructions for the Question Booklet

1. Do not open this question booklet until instructed.
2. After being instructed, write your name and examination registration number in space provided below, as printed on your examination voucher.
3. Course 1 is on pages 1-13, and Course 2 is on pages 15-27.
4. If your question booklet is missing any pages, raise your hand.
5. You may write notes and calculations in the question booklet.

III Instructions for how to answer the questions

1. You must mark your answers on the answer sheet with an HB pencil.
2. Each letter **A**, **B**, **C**, $\cdots$ in the questions represents a numeral (from 0 to 9) or the minus sign (−). When you mark your answers, fill in the oval completely for each letter in the corresponding row of the answer sheet (mark-sheet).
3. Sometimes an answer such as $\boxed{\text{A}}$ or $\boxed{\text{BC}}$ is used later in the question. In such a case, the symbol is shaded when it is used later, as $\boxed{\text{A}}$ or $\boxed{\text{BC}}$.

Note the following:

(1)Reduce square roots ($\sqrt{\ }$) as much as possible.
(Example : Express $\sqrt{32}$ as $4\sqrt{2}$, not as $2\sqrt{8}$ or $\sqrt{32}$.)

(2)For fractions, attach the minus sign to the numerator, and reduce the fraction to its lowest terms.
(Example : Substitute $\frac{1}{3}$ for $\frac{2}{6}$. Also simplify as follows :
$-\frac{2}{\sqrt{6}} = \frac{-2\sqrt{6}}{6} = \frac{-\sqrt{6}}{3}$. Then apply $\frac{-\sqrt{6}}{3}$ to the answer.)

(3)If your answer to $\dfrac{\boxed{\text{A}}\sqrt{\boxed{\text{B}}}}{\boxed{\text{C}}}$ is $\frac{-\sqrt{3}}{4}$, mark as shown below.

(4)If the answer to $\boxed{\text{DE}}\,x$ is $-x$, mark "−" for **D** and "1" for **E** as shown below.

A	●	⓪	①	②	③	④	⑤	⑥	⑦	⑧	⑨
B	⊖	⓪	①	②	●	④	⑤	⑥	⑦	⑧	⑨
C	⊖	⓪	①	②	③	●	⑤	⑥	⑦	⑧	⑨
D	●	⓪	①	②	③	④	⑤	⑥	⑦	⑧	⑨
E	⊖	⓪	●	②	③	④	⑤	⑥	⑦	⑧	⑨

4. Carefully read the instructions on the answer sheet, too.

※ Once you are instructed to start the examination, fill in your examination registration number and name.

Examination registration number			*					*						
Name														

Mathematics Course 1

(Basic Course)

（Course 2 begins on page 15）

Marking Your Choice of Course on the Answer Sheet

Choose to answer either Course 1 or Course 2.

If you choose Course 1, for example, circle the label “Course 1” and completely fill in the oval under the label on your answer sheet as shown in the example on the right.

< Example >

解答コース Course	
[✓ コース 1 Course 1]	コース 2 Course 2
●	○

If you do not correctly fill in the appropriate oval, your answers will not be graded.

$\boxed{\text{I}}$

Q 1 Set $P = 10a^2 + 14ab - 21bc - 15ca$.

(1) Factorizing P, we obtain

$$P = \left(\boxed{\textbf{A}}\,a + \boxed{\textbf{B}}\,b\right)\left(\boxed{\textbf{C}}\,a - \boxed{\textbf{D}}\,c\right).$$

(2) If $5a = \sqrt{6}$, $14b = \sqrt{2} + \sqrt{3} - \sqrt{6}$ and $15c = \sqrt{12} - \sqrt{8}$, then

$$P = \frac{\boxed{\textbf{E}} + \boxed{\textbf{F}}\sqrt{\boxed{\textbf{G}}}}{\boxed{\textbf{H}}}$$

and hence the greatest integer less than P is $\boxed{\textbf{I}}$.

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Q 2 There are two bags, A and B. Bag A contains four white balls and one red ball, and bag B contains two white balls and three red balls. Two balls are taken simultaneously out of bag A, then two balls are taken simultaneously out of bag B.

(1) The probability that two white balls are taken out of A, and one white ball and one red ball are taken out of B is $\dfrac{\boxed{\mathsf{J}}}{\boxed{\mathsf{KL}}}$.

(2) The probability that the four balls taken out consist of three white balls and one red ball is $\dfrac{\boxed{\mathsf{M}}}{\boxed{\mathsf{N}}}$.

(3) The probability that the four balls taken out all have the same color is $\dfrac{\boxed{\mathsf{O}}}{\boxed{\mathsf{PQ}}}$.

(4) The probability that of the four balls taken out, two or fewer are white balls is $\dfrac{\boxed{\mathsf{RS}}}{\boxed{\mathsf{TU}}}$.

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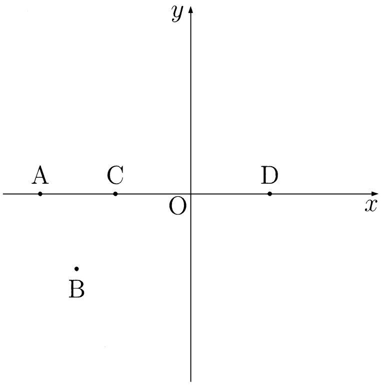
This is the end of the questions for Part $\boxed{\text{I}}$. Leave the answer spaces $\boxed{\textbf{V}}$ ～ $\boxed{\textbf{Z}}$ of Part $\boxed{\text{I}}$ blank.

II

Q 1 Consider the two parabolas

$$\ell: \quad y = ax^2 + 2bx + c$$

$$m: \quad y = (a+1)x^2 + 2(b+2)x + c + 3.$$



Four points A, B, C and D are assumed to be in the relative positions shown in the figure to the right. One of the two parabolas passes through the three points A, B and C, and the other one passes through the three points B, C and D.

(1) The parabola passing through the three points A, B and C is $\boxed{\text{A}}$. Here, for $\boxed{\text{A}}$ choose the correct answer from ⓪ or ①, just below.

⓪ parabola ℓ ① parabola m

(2) Since both parabolas ℓ and m pass through the two points B and C, the x-coordinates of B and C are the solutions of the quadratic equation

$$x^2 + \boxed{\text{B}}\,x + \boxed{\text{C}} = 0.$$

Hence, the x-coordinate of point B is $\boxed{\text{DE}}$, and the x-coordinate of point C is $\boxed{\text{FG}}$.

(3) In particular, we are to find the values of a, b and c when AB = BC and CO = OD.

Since the two points C and D are symmetric with respect to the y-axis, we have $b = \boxed{\text{H}}$. On the other hand, since AB = BC, the straight line $x = \boxed{\text{IJ}}$ is the axis of symmetry of $\boxed{\text{A}}$. Hence we have $a = -\dfrac{\boxed{\text{K}}}{\boxed{\text{L}}}$. And we have $c = \dfrac{\boxed{\text{M}}}{\boxed{\text{N}}}$.

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Q 2 We are to find the natural number a such that $3a+1$ is a divisor of a^2+5.

Set $b=3a+1$. Then we have

$$a^2+5=\frac{b^2-\boxed{\mathsf{O}}\,b+\boxed{\mathsf{PQ}}}{\boxed{\mathsf{R}}}. \qquad \cdots\cdots\cdots \quad ①$$

On the other hand, since b is a divisor of a^2+5, a^2+5 can be expressed as

$$a^2+5=bc \qquad \cdots\cdots\cdots \quad ②$$

for some natural number c. From ① and ②, we have

$$b\left(\boxed{\mathsf{S}}\,c-b+\boxed{\mathsf{T}}\right)=\boxed{\mathsf{UV}}.$$

This shows that b must also be one of the divisors of $\boxed{\mathsf{UV}}$. Of these, only $b=\boxed{\mathsf{WX}}$ is a number such that a is a natural number. Hence, $a=\boxed{\mathsf{YZ}}$.

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This is the end of the questions for Part $\boxed{\text{II}}$.

III

We have a triangle which has sides of the lengths 15, 19 and 23. We make it into an obtuse triangle by shortening each of its sides by x. What is the range of values that x can take?

First, since $15-x$, $19-x$ and $23-x$ can be the lengths of the sides of a triangle, it follows that

$$x < \boxed{\mathsf{AB}}.$$

In addition, such a triangle is an obtuse triangle only when x satisfies

$$x^2 - \boxed{\mathsf{CD}}\,x + \boxed{\mathsf{EF}} < 0.$$

By solving this quadratic inequality, we have

$$\boxed{\mathsf{G}} < x < \boxed{\mathsf{HI}}.$$

Hence, the range of x is

$$\boxed{\mathsf{J}} < x < \boxed{\mathsf{KL}}.$$

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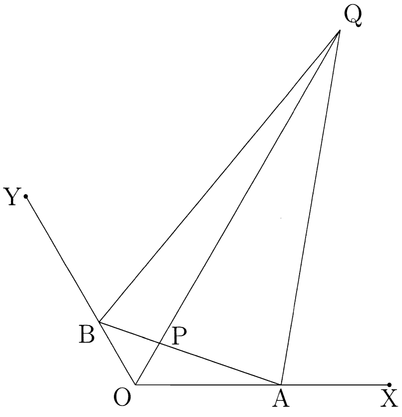
This is the end of the questions for Part $\boxed{\text{III}}$. Leave the answer spaces $\boxed{\textbf{M}}$ ～ $\boxed{\textbf{Z}}$ of Part $\boxed{\text{III}}$ blank.

IV

In the figure to the right, let

$$OA = 6, \quad OB = 3, \quad \angle AOB = 120^\circ,$$

and let the point Q denote the point of intersection of the bisector of $\angle XAB$ and the bisector of $\angle ABY$. Let P denote the point of intersection of segment AB and segment OQ. We are to find the length of segment PQ.



(1) First of all, we see that $AB = \boxed{\textbf{A}}\sqrt{\boxed{\textbf{B}}}$ and that the area of triangle OAB is $\dfrac{\boxed{\textbf{C}}\sqrt{\boxed{\textbf{D}}}}{\boxed{\textbf{E}}}$.

(2) For $\boxed{\textbf{F}}$ and $\boxed{\textbf{G}}$ in the following, choose the correct answer from among choices ⓪ ～ ④, just below.

⓪ AB ① AP ② AQ ③ BP ④ BQ

Since AQ is the bisector of the exterior angle of $\angle A$ of triangle OAP and BQ is the bisector of the exterior angle of $\angle B$ of triangle OBP, we have

$$\begin{aligned} OQ : PQ &= OA : \boxed{\textbf{F}} \\ &= OB : \boxed{\textbf{G}}. \end{aligned}$$

Hence we obtain $OA : OB = \boxed{\textbf{F}} : \boxed{\textbf{G}}$.

(3) Thus we see $AP = \boxed{\textbf{H}}\sqrt{\boxed{\textbf{I}}}$. Since $\angle AOP = \boxed{\textbf{JK}}^\circ$, we have $OP = \boxed{\textbf{L}}$.

Hence we have $PQ = \boxed{\textbf{M}} + \boxed{\textbf{N}}\sqrt{\boxed{\textbf{O}}}$.

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This is the end of the questions for Part $\boxed{\text{IV}}$.

Leave the answer spaces $\boxed{\textbf{P}}$ ～ $\boxed{\textbf{Z}}$ of Part $\boxed{\text{IV}}$ blank.

This is the end of the questions for Course 1. Leave the answer spaces for Part $\boxed{\text{V}}$ blank.

Please check once more that you have properly marked your course number as "Course 1" on your answer sheet.

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Mathematics Course 2

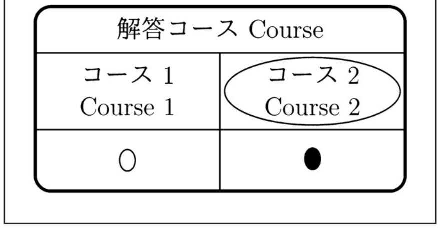
(Advanced Course)

Marking Your Choice of Course on the Answer Sheet

Choose to answer either Course 1 or Course 2.

If you choose Course 2, for example, circle the label "Course 2" and completely fill in the oval under the label on your answer sheet as shown in the example on the right.

< Example >



If you do not correctly fill in the appropriate oval, your answers will not be graded.

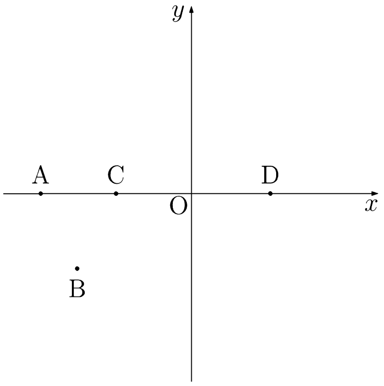
I

Q 1 Consider the two parabolas

$$\ell : \quad y = ax^2 + 2bx + c$$

$$m : \quad y = (a+1)x^2 + 2(b+2)x + c + 3.$$

Four points A, B, C and D are assumed to be in the relative positions shown in the figure to the right. One of the two parabolas passes through the three points A, B and C, and the other one passes through the three points B, C and D.



(1) The parabola passing through the three points A, B and C is $\boxed{\textbf{A}}$. Here, for $\boxed{\textbf{A}}$ choose the correct answer from ⓪ or ①, just below.

⓪ parabola ℓ ① parabola m

(2) Since both parabolas ℓ and m pass through the two points B and C, the x-coordinates of B and C are the solutions of the quadratic equation

$$x^2 + \boxed{\textbf{B}}\,x + \boxed{\textbf{C}} = 0.$$

Hence, the x-coordinate of point B is $\boxed{\textbf{DE}}$, and the x-coordinate of point C is $\boxed{\textbf{FG}}$.

(3) In particular, we are to find the values of a, b and c when AB = BC and CO = OD.

Since the two points C and D are symmetric with respect to the y-axis, we have $b = \boxed{\textbf{H}}$. On the other hand, since AB = BC, the straight line $x = \boxed{\textbf{IJ}}$ is the axis of symmetry of $\boxed{\textbf{A}}$. Hence we have $a = -\dfrac{\boxed{\textbf{K}}}{\boxed{\textbf{L}}}$. And we have $c = \dfrac{\boxed{\textbf{M}}}{\boxed{\textbf{N}}}$.

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Q 2 There are two bags, A and B. Bag A contains four white balls and one red ball, and bag B contains two white balls and three red balls. Two balls are taken simultaneously out of bag A, then two balls are taken simultaneously out of bag B.

(1) The probability that two white balls are taken out of A, and one white ball and one red ball are taken out of B is $\dfrac{\boxed{\mathbf{O}}}{\boxed{\mathbf{PQ}}}$.

(2) The probability that the four balls taken out consist of three white balls and one red ball is $\dfrac{\boxed{\mathbf{R}}}{\boxed{\mathbf{S}}}$.

(3) The probability that the four balls taken out all have the same color is $\dfrac{\boxed{\mathbf{T}}}{\boxed{\mathbf{UV}}}$.

(4) The probability that of the four balls taken out, two or fewer are white balls is $\dfrac{\boxed{\mathbf{WX}}}{\boxed{\mathbf{YZ}}}$.

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This is the end of the questions for Part $\boxed{\text{I}}$.

II

Q 1 Let $\overrightarrow{a}$ and $\overrightarrow{b}$ be two vectors such that $\left|\overrightarrow{a}\right| = 1$, $\left|\overrightarrow{b}\right| = 2$ and the angle formed by $\overrightarrow{a}$ and $\overrightarrow{b}$ is 60°. Set $\overrightarrow{u} = x\,\overrightarrow{a} + \overrightarrow{b}$ and $\overrightarrow{v} = x\,\overrightarrow{a} - \overrightarrow{b}$ for a real number x. When $x > 1$, we are to find the value of x such that the angle formed by $\overrightarrow{u}$ and $\overrightarrow{v}$ is 30°. In the following, $\overrightarrow{u} \cdot \overrightarrow{v}$ denotes the inner product of $\overrightarrow{u}$ and $\overrightarrow{v}$, and $\overrightarrow{a} \cdot \overrightarrow{b}$ denotes the inner product of $\overrightarrow{a}$ and $\overrightarrow{b}$.

First of all, since the angle formed by $\overrightarrow{u}$ and $\overrightarrow{v}$ is 30°, we obtain

$$\left(\overrightarrow{u} \cdot \overrightarrow{v}\right)^2 = \frac{\boxed{\textsf{A}}}{\boxed{\textsf{B}}} \left|\overrightarrow{u}\right|^2 \left|\overrightarrow{v}\right|^2.$$

When we express this equation in terms of x, noting $\overrightarrow{a} \cdot \overrightarrow{b} = \boxed{\textsf{C}}$, we have

$$x^4 - \boxed{\textsf{DE}}\,x^2 + \boxed{\textsf{FG}} = 0.$$

By transforming this, we also have

$$\left(x^2 - \boxed{\textsf{H}}\right)^2 = \left(\boxed{\textsf{I}}\,x\right)^2.$$

When this is solved for x, we obtain

$$x = \boxed{\textsf{J}} + \sqrt{\boxed{\textsf{KL}}},$$

noting $x > 1$.

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Q 2 In a complex number plane, consider the complex numbers z such that z^3 is a real number.

(1) Let C be the figure formed by the set of complex numbers $z = x + iy$ satisfying the above condition. Since the arguments of the complex numbers z satisfy

$$\arg z = \frac{\pi}{\boxed{\mathbf{M}}}k \qquad (k:\ \text{integer}),$$

figure C consists of three straight lines represented in terms of x and y by the equations

$$y = \boxed{\mathbf{N}}, \quad y = \sqrt{\boxed{\mathbf{O}}}\,x, \quad y = -\sqrt{\boxed{\mathbf{P}}}\,x.$$

(2) Suppose that on C there exists only one complex number z satisfying $|z - 1 - i| = r$. Then the value of r is

$$r = \frac{\sqrt{\boxed{\mathbf{Q}}} - \boxed{\mathbf{R}}}{\boxed{\mathbf{S}}},$$

and the value of z is

$$z = \frac{\boxed{\mathbf{T}} + \sqrt{\boxed{\mathbf{U}}}}{\boxed{\mathbf{V}}}\left(1 + \sqrt{\boxed{\mathbf{W}}}\,i\right).$$

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This is the end of the questions for Part $\boxed{\text{II}}$. Leave the answer spaces $\boxed{\textbf{X}}$ ~ $\boxed{\textbf{Z}}$ of Part $\boxed{\text{II}}$ blank.

III

We are to find the range of the values of a real number t such that the maximum value of the cubic function

$$f(x) = \frac{1}{3}x^3 - \frac{t+2}{2}x^2 + 2tx + \frac{2}{3}$$

over the interval $x \leqq 4$ is greater than 6.

First of all, since the derivative of $f(x)$ is

$$f'(x) = (x - \boxed{\textsf{A}})(x - t),$$

we consider the problem by dividing the range of the values of t as follows:

(i) When $t > \boxed{\textsf{A}}$, $f(x)$ has a local maximum at $x = \boxed{\textsf{A}}$ and a local minimum at $x = t$. Since $f(4) = \boxed{\textsf{B}}$, we only have to find the range of the values of t satisfying $f(\boxed{\textsf{A}}) > 6$.

(ii) When $t = \boxed{\textsf{A}}$, the maximum value of $f(x)$ over the interval $x \leqq 4$ is $f(\boxed{\textsf{C}}) = \boxed{\textsf{D}}$, and hence the condition is not satisfied.

(iii) When $t < \boxed{\textsf{A}}$, $f(x)$ has a local maximum at $x = t$ and a local minimum at $x = \boxed{\textsf{A}}$. Since $f(4) = \boxed{\textsf{B}}$, we only have to find the range of the values of t satisfying $f(t) > 6$.

Here, we note

$$f(t) - 6 = -\frac{1}{6}\left(t + \boxed{\textsf{E}}\right)\left(t - \boxed{\textsf{F}}\right)^2.$$

From the above, the range of the values of t is

$$t > \frac{\boxed{\textsf{GH}}}{\boxed{\textsf{I}}} \quad \text{or} \quad t < \boxed{\textsf{JK}}.$$

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This is the end of the questions for Part $\boxed{\text{III}}$. Leave the answer spaces $\boxed{\textbf{L}}$ ~ $\boxed{\textbf{Z}}$ of Part $\boxed{\text{III}}$ blank.

IV

Consider the function

$$f(x) = \frac{\sin x}{3 - 2\cos x} \qquad (\,0 \leqq x \leqq \pi\,).$$

(1) The derivative of $f(x)$ is

$$f'(x) = \frac{\boxed{\mathbf{A}}\cos x - \boxed{\mathbf{B}}}{\left(\boxed{\mathbf{C}} - \boxed{\mathbf{D}}\cos x\right)^2}\,.$$

Let α be the value of x at which $f(x)$ has a local extremum. Then we have

$$\cos\alpha = \frac{\boxed{\mathbf{E}}}{\boxed{\mathbf{F}}}\,.$$

(2) The portion of the plane bounded by the graph of the function $y = f(x)$ and the x-axis is divided into two parts by the straight line $x = \alpha$. Let S_1 be the area of the part located on the left side of the line. Then we have

$$S_1 = \int_{\frac{\boxed{\mathbf{G}}}{\boxed{\mathbf{H}}}}^{\boxed{\mathbf{I}}} \frac{dt}{\boxed{\mathbf{J}} - \boxed{\mathbf{K}}\,t} = \frac{\boxed{\mathbf{L}}}{\boxed{\mathbf{M}}}\log\frac{\boxed{\mathbf{N}}}{\boxed{\mathbf{O}}}\,.$$

Let S_2 be the area of the part located on the right side. We have

$$S_2 = \frac{\boxed{\mathbf{P}}}{2}\log\boxed{\mathbf{Q}}\,.$$

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This is the end of the questions for Part $\boxed{\text{IV}}$.

Leave the answer spaces **R** ～ **Z** of Part $\boxed{\text{IV}}$ blank.

This is the end of the questions for Course 2. Leave the answer spaces for Part $\boxed{\text{V}}$ blank.

Please check once more that you have properly marked your course number as "Course 2" on your answer sheet.

Do not take this question booklet out of the room.