2020 Examination for Japanese University Admission for International Students

Mathematics (80 min.)

【Course 1 (Basic), Course 2 (Advanced)】

※ Choose one of these courses and answer its questions only.

I Rules of Examination

1. Do not leave the room without proctor's permission.
2. Do not take this question booklet out of the room.

II Instructions for the Question Booklet

1. Do not open this question booklet until instructed.
2. After being instructed, write your name and examination registration number in space provided below, as printed on your examination voucher.
3. Course 1 is on pages 1-13, and Course 2 is on pages 15-27.
4. If your question booklet is missing any pages, raise your hand.
5. You may write notes and calculations in the question booklet.

III Instructions for how to answer the questions

1. You must mark your answers on the answer sheet with an HB pencil.
2. Each letter **A**, **B**, **C**, $\cdots$ in the questions represents a numeral (from 0 to 9) or the minus sign ($-$). When you mark your answers, fill in the oval completely for each letter in the corresponding row of the answer sheet (mark-sheet).
3. Sometimes an answer such as $\boxed{\text{A}}$ or $\boxed{\text{BC}}$ is used later in the question. In such a case, the symbol is shaded when it is used later, as $\boxed{\text{A}}$ or $\boxed{\text{BC}}$.

Note the following :

(1) Reduce square roots ($\sqrt{\ }$) as much as possible.
(Example: Express $\sqrt{32}$ as $4\sqrt{2}$, not as $2\sqrt{8}$ or $\sqrt{32}$.)

(2) For fractions, attach the minus sign to the numerator, and reduce the fraction to its lowest terms.
(Example: Substitute $\frac{1}{3}$ for $\frac{2}{6}$. Also simplify as follows:
$-\frac{2}{\sqrt{6}} = \frac{-2\sqrt{6}}{6} = \frac{-\sqrt{6}}{3}$. Then apply $\frac{-\sqrt{6}}{3}$ to the answer.)

(3) If your answer to $\dfrac{\boxed{\text{A}}\sqrt{\boxed{\text{B}}}}{\boxed{\text{C}}}$ is $\frac{-\sqrt{3}}{4}$, mark as shown below.

(4) If the answer to $\boxed{\text{DE}}\,x$ is $-x$, mark "$-$" for **D** and "1" for **E** as shown below.

A	●	⓪	①	②	③	④	⑤	⑥	⑦	⑧	⑨
B	⊖	⓪	①	②	●	④	⑤	⑥	⑦	⑧	⑨
C	⊖	⓪	①	②	③	●	⑤	⑥	⑦	⑧	⑨
D	●	⓪	①	②	③	④	⑤	⑥	⑦	⑧	⑨
E	⊖	⓪	●	②	③	④	⑤	⑥	⑦	⑧	⑨

4. Carefully read the instructions on the answer sheet, too.

※ Once you are instructed to start the examination, fill in your examination registration number and name.

Examination registration number			*					*						
Name														

Mathematics Course 1

(Basic Course)

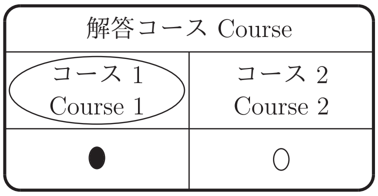
(Course 2 begins on page 15)

Marking Your Choice of Course on the Answer Sheet

Choose to answer <u>either</u> Course 1 or Course 2.

If you choose Course 1, for example, circle the label “Course 1” and completely fill in the oval under the label on your answer sheet as shown in the example on the right.

< Example >



<u>**If you do not correctly fill in the appropriate oval, your answers will not be graded.**</u>

I

Q 1 Let a be a positive constant. When we move the graph of the quadratic function $y = \frac{1}{4}x^2$ by parallel translation, the resulting parabola and the x-axis intersect at $(-2a,\ 0)$ and $(4a,\ 0)$. Let us consider the equation $y = f(x)$ of this parabola.

(1) The function $f(x)$ can be expressed as

$$f(x) = \frac{\boxed{\mathsf{A}}}{\boxed{\mathsf{B}}}\left(x - \boxed{\mathsf{C}}\,a\right)\left(x + \boxed{\mathsf{D}}\,a\right).$$

(2) The range of values of x such that the value of $y = f(x)$ is less than or equal to $10a^2$ can be obtained by solving the inequality

$$x^2 - \boxed{\mathsf{E}}\,ax - \boxed{\mathsf{FG}}\,a^2 \leqq 0,$$

and it is $-\boxed{\mathsf{H}}\,a \leqq x \leqq \boxed{\mathsf{I}}\,a$.

(3) Suppose that the length of the segment between the intersections of the straight line $y = 10a$ and the graph of $y = f(x)$ is 10. Since $\boxed{\mathsf{J}}\sqrt{\boxed{\mathsf{K}}\,a^2 + \boxed{\mathsf{LM}}\,a} = 10$, we see that the value of a is $\dfrac{\boxed{\mathsf{N}}}{\boxed{\mathsf{O}}}$.

- memo -

Q 2 There is a staircase of 10 steps which we are to climb. We can go up one step at a time or two steps at a time, but we have to use each method at least once.

(1) Suppose we can go up two steps at a time twice or more in a row.

(i) If we climb the staircase going up two steps at a time just 3 times, we will go up one step at a time just $\boxed{\mathsf{P}}$ times, and there are $\boxed{\mathsf{QR}}$ different ways of climbing the staircase.

(ii) If we can go up two steps at a time twice or more in a row, there are altogether $\boxed{\mathsf{ST}}$ different ways of climbing the staircase.

(2) Suppose we cannot go up two steps at a time twice or more in a row.

(i) If we climb the staircase going up two steps at a time just twice, we will go up one step at a time just $\boxed{\mathsf{U}}$ times, and there are $\boxed{\mathsf{VW}}$ different ways of climbing the staircase.

(ii) If we cannot go up two steps at a time twice or more in a row, there are altogether $\boxed{\mathsf{XY}}$ different ways of climbing the staircase.

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This is the end of the questions for Part $\boxed{\text{I}}$. Leave the answer space $\boxed{\textsf{Z}}$ of Part $\boxed{\text{I}}$ blank.

II

Q 1 Let m and n be positive integers satisfying $0 < m - n\sqrt{2} < 1$. Denote the integral part of $\left(m + n\sqrt{2}\right)^3$ by a and the fractional part by b.

(1) We are to prove that a is an odd number and $\left(m - n\sqrt{2}\right)^3 = 1 - b$.

If $\left(m + n\sqrt{2}\right)^3 = p + q\sqrt{2}$, where p and q are integers, then

$$p = m^3 + \boxed{\text{A}}\,mn^2, \quad q = \boxed{\text{B}}\,m^2 n + \boxed{\text{C}}\,n^3.$$

Thus, we see that $\left(m - n\sqrt{2}\right)^3 = p - q\sqrt{2}$.

Furthermore, the integral part of $\left(m - n\sqrt{2}\right)^3$ is $\boxed{\text{D}}$. When we denote its fractional part by c, the following two equations hold:

$$\begin{cases} p + q\sqrt{2} = a + b \\ p - q\sqrt{2} = c. \end{cases}$$

From these we obtain

$$\boxed{\text{E}}\,p - a = b + c.$$

Here, since the left side is an integer and the range of values which the right side takes is $\boxed{\text{F}} < b + c < \boxed{\text{G}}$, we see that

$$b + c = \boxed{\text{H}}.$$

Hence we see that $a = \boxed{\text{E}}\,p - \boxed{\text{H}}$, which shows that a is an odd number and that $\left(m - n\sqrt{2}\right)^3 = 1 - b$.

(2) Let us find the values of m and n when $a = 197$.

Since $a = 197$, we see that $p = \boxed{\text{IJ}}$, that is, $m^3 + \boxed{\text{A}}\,mn^2 = \boxed{\text{IJ}}$. The positive integers m and n satisfying this equation are

$$m = \boxed{\text{K}}, \quad n = \boxed{\text{L}}.$$

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Q 2 Let a be a real number satisfying $a \geqq 0$. We are to express the maximum value M of the function $f(x) = |x^2 - 2x|$ on the range $a \leqq x \leqq a+1$ in terms of a. Furthermore, we are to find the minimum value of M over the range $a \geqq 0$.

(1) The function $f(x)$ can be expressed without using the absolute value symbol as follows:

when $x \leqq \boxed{\mathbf{M}}$ or $x \geqq \boxed{\mathbf{N}}$, then $f(x) = x^2 - 2x$;

when $\boxed{\mathrm{M}} < x < \boxed{\mathrm{N}}$, then $f(x) = -x^2 + 2x$.

Hence, the maximum value of $f(x)$ on $a \leqq x \leqq a+1$ is the following:

when $0 \leqq a \leqq \boxed{\mathbf{O}}$, then $M = \boxed{\mathbf{P}}$;

when $\boxed{\mathrm{O}} < a \leqq \dfrac{\boxed{\mathbf{Q}} + \sqrt{\boxed{\mathbf{R}}}}{\boxed{\mathbf{S}}}$, then $M = -a^2 + \boxed{\mathbf{T}}\,a$;

when $a > \dfrac{\boxed{\mathrm{Q}} + \sqrt{\boxed{\mathrm{R}}}}{\boxed{\mathrm{S}}}$, then $M = a^2 - \boxed{\mathbf{U}}$.

(2) The minimum value of M over the range $a \geqq 0$ is $\dfrac{\sqrt{\boxed{\mathbf{V}}}}{\boxed{\mathbf{W}}}$.

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This is the end of the questions for Part $\boxed{\text{II}}$. Leave the answer spaces $\boxed{\mathbf{X}}$ ～ $\boxed{\mathbf{Z}}$ of Part $\boxed{\text{II}}$ blank.

III

Consider integers a and b satisfying the equation

$$14a + 9b = 147. \qquad \cdots\cdots\cdots \quad ①$$

(1) We are to find the positive integers a and b satisfying equation ①.

Since

$$14a = \boxed{\mathsf{A}}\left(\boxed{\mathsf{BC}} - \boxed{\mathsf{D}}\,b\right) \quad \text{and} \quad 9b = \boxed{\mathsf{E}}\left(\boxed{\mathsf{FG}} - \boxed{\mathsf{H}}\,a\right),$$

a is a multiple of $\boxed{\mathsf{A}}$, and b is a multiple of $\boxed{\mathsf{E}}$.

So, when we set $a = \boxed{\mathsf{A}}\,m$ and $b = \boxed{\mathsf{E}}\,n$, where m and n are integers, from ① we have

$$\boxed{\mathsf{I}}\,m + \boxed{\mathsf{J}}\,n = \boxed{\mathsf{K}}.$$

Since the positive integers m and n satisfying this are

$$m = \boxed{\mathsf{L}} \quad \text{and} \quad n = \boxed{\mathsf{M}},$$

we obtain

$$a = \boxed{\mathsf{N}} \quad \text{and} \quad b = \boxed{\mathsf{O}}.$$

(2) We are to find the solutions a and b of equation ① satisfying $0 < a + b < 5$.

Since $14 \times \boxed{\mathsf{N}} + 9 \times \boxed{\mathsf{O}} = 147$, from this equality and ① we have

$$14(a - \boxed{\mathsf{N}}) = 9(\boxed{\mathsf{O}} - b).$$

Since 14 and 9 are relatively prime, a and b can be expressed in terms of an integer k as

$$a = \boxed{\mathsf{P}}\,k + \boxed{\mathsf{Q}}, \qquad b = -\boxed{\mathsf{RS}}\,k + \boxed{\mathsf{T}}.$$

Since $0 < a + b < 5$, we have $k = \boxed{\mathsf{U}}$, and we obtain

$$a = \boxed{\mathsf{VW}}, \qquad b = -\boxed{\mathsf{XY}}.$$

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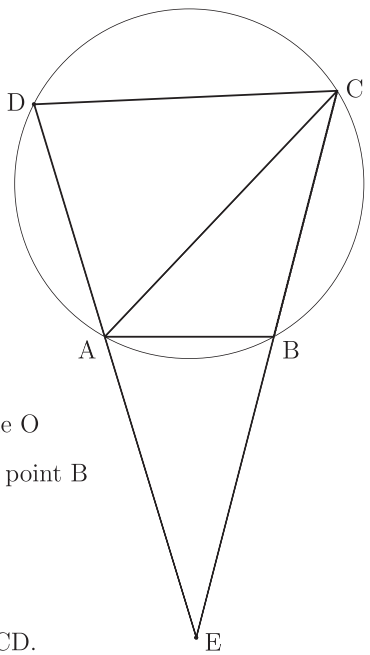
This is the end of the questions for Part $\boxed{\text{III}}$. Leave the answer space $\boxed{\mathsf{Z}}$ of Part $\boxed{\text{III}}$ blank.

IV



Consider a triangle ABC and its circumscribed circle O, where the lengths of the three sides of the triangle are

$$\mathrm{AB} = 2, \quad \mathrm{BC} = 3, \quad \mathrm{CA} = 4.$$

Below, the area of a triangle such as PQR is expressed as $\triangle\mathrm{PQR}$.

(1) We see that $\cos\angle\mathrm{ABC} = \dfrac{\boxed{\mathbf{AB}}}{\boxed{\mathbf{C}}}$.

(2) Let us take a point D on the circumference of circle O such that it is on the opposite side of the circle from point B with respect to AC and

$$\frac{\triangle\mathrm{ABD}}{\triangle\mathrm{BCD}} = \frac{8}{15}. \quad \cdots\cdots\cdots \quad ①$$

We are to find the lengths of line segments AD and CD.

First, since

$$\angle\mathrm{BAD} = \boxed{\mathbf{DEF}}^{\circ} - \angle\mathrm{BCD},$$

we have $\sin\angle\mathrm{BAD} = \sin\angle\mathrm{BCD}$. Hence from ① we have

$$\frac{\mathrm{AD}}{\mathrm{CD}} = \frac{\boxed{\mathbf{G}}}{\boxed{\mathbf{H}}},$$

so we set $\mathrm{AD} = \boxed{\mathrm{G}}\,k$ and $\mathrm{CD} = \boxed{\mathrm{H}}\,k$, where k is a positive number. Furthermore, since

$$\angle\mathrm{ADC} = \boxed{\mathbf{IJK}}^{\circ} - \angle\mathrm{ABC},$$

we have $\cos\angle\mathrm{ADC} = \dfrac{\boxed{\mathbf{L}}}{\boxed{\mathbf{M}}}$. Hence, we obtain $k = \dfrac{\boxed{\mathbf{N}}}{\sqrt{\boxed{\mathbf{OP}}}}$, and then

$$\mathrm{AD} = \frac{\boxed{\mathbf{QR}}\sqrt{\boxed{\mathrm{OP}}}}{\boxed{\mathrm{OP}}}, \quad \mathrm{CD} = \frac{\boxed{\mathbf{ST}}\sqrt{\boxed{\mathrm{OP}}}}{\boxed{\mathrm{OP}}}.$$

(3) When we denote the point of intersection of the straight line DA and the straight line CB by E, we have

$$\frac{\triangle\mathrm{ABE}}{\triangle\mathrm{CDE}} = \frac{\boxed{\mathbf{UV}}}{\boxed{\mathbf{WXY}}}.$$

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This is the end of the questions for Part $\boxed{\text{IV}}$. Leave the answer space $\boxed{\textbf{Z}}$ of Part $\boxed{\text{IV}}$ blank.

This is the end of the questions for Course 1. Leave the answer spaces for Part $\boxed{\text{V}}$ blank.

Please check once more that you have properly marked your course number as "Course 1" on your answer sheet.

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Mathematics Course 2

(Advanced Course)

Marking Your Choice of Course on the Answer Sheet

Choose to answer <u>either</u> Course 1 or Course 2.

If you choose Course 2, for example, circle the label “Course 2” and completely fill in the oval under the label on your answer sheet as shown in the example on the right.

< Example >

解答コース Course	
コース 1 Course 1	コース 2 Course 2
○	●

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Q 1 Let a be a positive constant. When we move the graph of the quadratic function $y = \frac{1}{4}x^2$ by parallel translation, the resulting parabola and the x-axis intersect at $(-2a,\ 0)$ and $(4a,\ 0)$. Let us consider the equation $y = f(x)$ of this parabola.

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Q 2 There is a staircase of 10 steps which we are to climb. We can go up one step at a time or two steps at a time, but we have to use each method at least once.

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(ii) If we cannot go up two steps at a time twice or more in a row, there are altogether $\boxed{\mathbf{XY}}$ different ways of climbing the staircase.

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This is the end of the questions for Part $\boxed{\text{I}}$. Leave the answer space $\boxed{\textsf{Z}}$ of Part $\boxed{\text{I}}$ blank.

II

Q 1 Let $\{a_n\}$ be a sequence such that the sum S_n of the terms from the first term to the n-th term is

$$S_n = \frac{n^2 - 17n}{4},$$

and let $\{b_n\}$ be the sequence defined by

$$b_n = a_n \cdot a_{n+5} \qquad (n = 1, 2, 3, \cdots).$$

(1) For $\boxed{\textbf{A}}$ ~ $\boxed{\textbf{C}}$ in the following sentences, choose the correct answer from among choices ⓪ ~ ⑨ below.

Let us find the sum T_n of the terms of sequence $\{b_n\}$ from the first term to the n-th term.

Since $a_n = \boxed{\textbf{A}}$, we have $b_n = \boxed{\textbf{B}}$. Hence we obtain

$$T_n = \boxed{\textbf{C}}.$$

⓪ $\dfrac{n-7}{2}$ ① $\dfrac{n-9}{2}$ ② $\dfrac{n-11}{2}$

③ $\dfrac{n^2-12n+27}{4}$ ④ $\dfrac{n^2-13n+36}{4}$ ⑤ $\dfrac{n^2-14n+45}{4}$

⑥ $\dfrac{n(n^2-17n+83)}{12}$ ⑦ $\dfrac{n(n^2-17n+89)}{12}$ ⑧ $\dfrac{n(n^2-18n+83)}{12}$

⑨ $\dfrac{n(n^2-18n+89)}{12}$

(This question is continued on the next page.)

(2) Next, let us find the minimum value of T_n.

When $n \leqq$ $\boxed{\mathsf{D}}$ or $\boxed{\mathsf{EF}}$ $\leqq n$, we see that $b_n > 0$. On the other hand, when $\boxed{\mathsf{G}}$ $\leqq n \leqq$ $\boxed{\mathsf{H}}$, we see that $b_n < 0$.

Hence T_n is minimized at $n =$ $\boxed{\mathsf{I}}$, $n =$ $\boxed{\mathsf{J}}$ and $n =$ $\boxed{\mathsf{K}}$, and its minimum value is $\boxed{\mathsf{L}}$. (Answer in the order such that $\boxed{\mathsf{I}} < \boxed{\mathsf{J}} < \boxed{\mathsf{K}}$.)

Q 2 Answer the following questions.

(1) When we express the complex number $8+8\sqrt{3}\,i$ in polar form, we have

$$\boxed{\mathsf{MN}}\left(\cos\frac{\pi}{\boxed{\mathsf{O}}}+i\sin\frac{\pi}{\boxed{\mathsf{P}}}\right).$$

(2) Consider the complex numbers z that satisfy $z^4=8+8\sqrt{3}\,i$ in the range $0 \leqq \arg z < 2\pi$.

We see that $|z| = \boxed{\mathsf{Q}}$. There are 4 such complex numbers z. When these are denoted by $z_1,\ z_2,\ z_3,\ z_4$ in the ascending order of their arguments, we have

$$\arg\frac{z_1z_2z_3}{z_4} = \frac{\pi}{\boxed{\mathsf{R}}}.$$

(3) Consider the complex numbers w that satisfy $w^8-16w^4+256=0$ in the range $0 \leqq \arg w < 2\pi$. There are 8 such complex numbers w. Let us denote them by $w_1,\ w_2,\ w_3,\ w_4,\ w_5,\ w_6,\ w_7,\ w_8$ in the ascending order of their arguments. Then four of these coincide with numbers $z_1,\ z_2,\ z_3,\ z_4$ in (2). That is,

$$w_{\boxed{\mathsf{S}}}=z_1,\quad w_{\boxed{\mathsf{T}}}=z_2,\quad w_{\boxed{\mathsf{U}}}=z_3,\quad w_{\boxed{\mathsf{V}}}=z_4.$$

Also, we have that $w_1w_8 = \boxed{\mathsf{W}}$ and $w_3w_4 = \boxed{\mathsf{XY}}\,i$.

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This is the end of the questions for Part $\boxed{\text{II}}$. Leave the answer space $\boxed{\mathsf{Z}}$ of Part $\boxed{\text{II}}$ blank.

III

Consider the function $f(x) = x^3 - 4x + 4$.

Let the straight line ℓ be the tangent to the graph of $y = f(x)$ at the point A$(-1, 7)$, and the straight line m be the tangent to the graph of $y = f(x)$ that passes through the point B$(0, -12)$. Also, let C be the point of intersection of ℓ and m. Let us denote the angle formed by ℓ and m at C by θ $\left(0 < \theta < \dfrac{\pi}{2}\right)$. We are to find $\tan\theta$.

(1) The derivative $f'(x)$ of $f(x)$ is

$$f'(x) = \boxed{\mathbf{A}}\,x^{\boxed{\mathbf{B}}} - \boxed{\mathbf{C}}.$$

Hence, the slope of ℓ is $\boxed{\mathbf{DE}}$, and the equation of ℓ is

$$y = \boxed{\mathrm{DE}}\,x + \boxed{\mathbf{F}}.$$

(2) Let us denote by a the x-coordinate of the tangent point of the graph of $y = f(x)$ and line m. Then the equation of m can be expressed in terms of a as

$$y = \left(\boxed{\mathbf{G}}\,a^{\boxed{\mathbf{H}}} - \boxed{\mathbf{I}}\right)x - \boxed{\mathbf{J}}\,a^{\boxed{\mathbf{K}}} + \boxed{\mathbf{L}}.$$

Since line m passes through point B$(0, -12)$, we see that $a = \boxed{\mathbf{M}}$, and the equation of m is

$$y = \boxed{\mathbf{N}}\,x - \boxed{\mathbf{OP}}.$$

Hence, the coordinates of point C, the intersection of ℓ and m, are $\left(\boxed{\mathbf{Q}}, \boxed{\mathbf{R}}\right)$.

(3) Let us denote by α the angle between the positive direction of the x-axis and line ℓ, and by β the angle between the positive direction of the x-axis and line m. Then we see that

$$\tan\alpha = \boxed{\mathbf{ST}}, \quad \tan\beta = \boxed{\mathbf{U}},$$

and hence

$$\tan\theta = \frac{\boxed{\mathbf{V}}}{\boxed{\mathbf{W}}}.$$

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This is the end of the questions for Part $\boxed{\text{III}}$. Leave the answer spaces $\boxed{\textbf{X}}$ ～ $\boxed{\textbf{Z}}$ of Part $\boxed{\text{III}}$ blank.

IV

Consider the function

$$f(x) = \sin x + \frac{\sin 2x}{2} + \frac{\sin 3x}{3}$$

on the interval $0 \leqq x \leqq \pi$. We are to show that $f(x) > 0$ on $0 < x < \pi$, and to find the area S of the region bounded by the graph of $y = f(x)$ and the x-axis.

(1) For $\boxed{\mathbf{K}}$, $\boxed{\mathbf{N}}$, $\boxed{\mathbf{Q}}$, $\boxed{\mathbf{R}}$ in the following sentences, choose the correct answer from the following two choices

⓪ increasing ① decreasing,

and for the other $\boxed{}$, enter the correct number.

When we differentiate $f(x)$, we have

$$f'(x) = \left(\boxed{\mathbf{A}}\cos^2 x - \boxed{\mathbf{B}}\right)\left(\boxed{\mathbf{C}}\cos x + \boxed{\mathbf{D}}\right).$$

Hence, over the range $0 \leqq x \leqq \pi$, there are three x's at which $f'(x) = 0$, and when they are arranged in ascending order, they are

$$x = \frac{\pi}{\boxed{\mathbf{E}}}, \quad \frac{\boxed{\mathbf{F}}}{\boxed{\mathbf{G}}}\pi, \quad \frac{\boxed{\mathbf{H}}}{\boxed{\mathbf{I}}}\pi.$$

Next, looking at whether $f(x)$ is increasing or decreasing, we see that:

on $0 \leqq x \leqq \dfrac{\pi}{\boxed{\mathbf{J}}}$, $f(x)$ is $\boxed{\mathbf{K}}$;

on $\dfrac{\pi}{\boxed{\mathrm{J}}} \leqq x \leqq \dfrac{\boxed{\mathbf{L}}}{\boxed{\mathbf{M}}}\pi$, $f(x)$ is $\boxed{\mathbf{N}}$;

on $\dfrac{\boxed{\mathrm{L}}}{\boxed{\mathrm{M}}}\pi \leqq x \leqq \dfrac{\boxed{\mathbf{O}}}{\boxed{\mathbf{P}}}\pi$, $f(x)$ is $\boxed{\mathbf{Q}}$;

on $\dfrac{\boxed{\mathrm{O}}}{\boxed{\mathrm{P}}}\pi \leqq x \leqq \pi$, $f(x)$ is $\boxed{\mathbf{R}}$.

Also, we have

$$f(0) = 0, \quad f(\pi) = 0, \quad f\left(\frac{\boxed{\mathrm{L}}}{\boxed{\mathrm{M}}}\pi\right) = \frac{\sqrt{\boxed{\mathbf{S}}}}{\boxed{\mathbf{T}}} > 0.$$

Hence we see that $f(x) > 0$ on $0 < x < \pi$.

(This question is continued on the next page.)

(2) The area S of the region bounded by the graph of $y = f(x)$ and the x-axis is

$$S = \frac{\boxed{\mathsf{UV}}}{\boxed{\mathsf{W}}}.$$

This is the end of the questions for Part $\boxed{\text{IV}}$.

Leave the answer spaces $\boxed{\mathsf{X}}$ ~ $\boxed{\mathsf{Z}}$ of Part $\boxed{\text{IV}}$ blank.

This is the end of the questions for Course 2. Leave the answer spaces for Part $\boxed{\text{V}}$ blank.

Please check once more that you have properly marked your course number as "Course 2" on your answer sheet.

Do not take this question booklet out of the room.