2018 Examination for Japanese University Admission for International Students

Mathematics (80 min.)

【Course 1 (Basic), Course 2 (Advanced)】

※ Choose <u>one</u> of these courses and answer its questions only.

I Rules of Examination

1. Do not leave the room without proctor's permission.
2. Do not take this question booklet out of the room.

II Instructions for the Question Booklet

1. Do not open this question booklet until instructed.
2. After being instructed, write your name and examination registration number in space provided below, as printed on your examination voucher.
3. Course 1 is on pages 1-14, and Course 2 is on pages 15-27.
4. If your question booklet is missing any pages, raise your hand.
5. You may write notes and calculations in the question booklet.

III Instructions for how to answer the questions

1. You must mark your answers on the answer sheet with an HB pencil.
2. Each letter **A**, **B**, **C**, $\cdots$ in the questions represents a numeral (from 0 to 9) or the minus sign $(-)$. When you mark your answers, fill in the oval completely for each letter in the corresponding row of the answer sheet (mark-sheet).
3. Sometimes an answer such as $\boxed{\text{A}}$ or $\boxed{\text{BC}}$ is used later in the question. In such a case, the symbol is shaded when it is used later, as $\boxed{\text{A}}$ or $\boxed{\text{BC}}$.

Note the following :

(1) Reduce square roots ($\sqrt{\ }$) as much as possible.
(Example: Express $\sqrt{32}$ as $4\sqrt{2}$, not as $2\sqrt{8}$ or $\sqrt{32}$.)

(2) For fractions, attach the minus sign to the numerator, and reduce the fraction to its lowest terms.
(Example: Substitute $\frac{1}{3}$ for $\frac{2}{6}$. Also simplify as follows:
$-\frac{2}{\sqrt{6}} = \frac{-2\sqrt{6}}{6} = \frac{-\sqrt{6}}{3}$. Then apply $\frac{-\sqrt{6}}{3}$ to the answer.)

(3) If your answer to $\dfrac{\boxed{\text{A}}\sqrt{\boxed{\text{B}}}}{\boxed{\text{C}}}$ is $\frac{-\sqrt{3}}{4}$, mark as shown below.

(4) If the answer to $\boxed{\text{DE}}\,x$ is $-x$, mark "$-$" for **D** and "1" for **E** as shown below.

A	●	⓪	①	②	③	④	⑤	⑥	⑦	⑧	⑨
B	⊖	⓪	①	②	●	④	⑤	⑥	⑦	⑧	⑨
C	⊖	⓪	①	②	③	●	⑤	⑥	⑦	⑧	⑨
D	●	⓪	①	②	③	④	⑤	⑥	⑦	⑧	⑨
E	⊖	⓪	●	②	③	④	⑤	⑥	⑦	⑧	⑨

4. Carefully read the instructions on the answer sheet, too.

※ Once you are instructed to start the examination, fill in your examination registration number and name.

Examination registration number			*					*						
Name														

Mathematics Course 1

(Basic Course)

(Course 2 begins on page 15)

Marking Your Choice of Course on the Answer Sheet

Choose to answer either Course 1 or Course 2.

If you choose Course 1, for example, circle the label "Course 1" and completely fill in the oval under the label on your answer sheet as shown in the example on the right.

< Example >

解答コース Course	
コース 1 Course 1	コース 2 Course 2
●	○

If you do not correctly fill in the appropriate oval, your answers will not be graded.

I

Q 1 Let us consider the maximum value M and the minimum value m of the quadratic function

$$f(x) = x^2 - 2(a+1)x + 2a^2$$

over $0 \leqq x \leqq 2$, where a is a constant and $0 \leqq a \leqq 3$.

(1) The coordinates of the vertex of the graph of $y = f(x)$ are

$$\left(a + \boxed{\mathbf{A}},\ a^2 - \boxed{\mathbf{B}}\,a - \boxed{\mathbf{C}}\right).$$

(2) For $\boxed{\mathbf{D}}$ ～ $\boxed{\mathbf{H}}$ in the following sentences, choose the correct answers from among choices ⓪ ～ ⑨ below.

Let us find the maximum value M and the minimum value m according to the position of the axis of symmetry. We have that

if $0 \leqq a < \boxed{\mathbf{D}}$, then

$$M = \boxed{\mathbf{E}}, \quad m = \boxed{\mathbf{F}};$$

if $\boxed{\mathrm{D}} \leqq a \leqq 3$, then

$$M = \boxed{\mathbf{G}}, \quad m = \boxed{\mathbf{H}}.$$

⓪ 0 ① 1 ② 2 ③ 3

④ $a^2 - 2a$ ⑤ $a^2 - 2a - 1$ ⑥ $2a^2$

⑦ $2a^2 - 2a - 1$ ⑧ $2a^2 - 4a$ ⑨ $2a^2 - 6a + 3$

(3) Thus, m is maximized at $a = \boxed{\mathbf{I}}$ and the value of m then is $\boxed{\mathbf{J}}$. Also, m is minimized at $a = \boxed{\mathbf{K}}$ and the value of m then is $\boxed{\mathbf{LM}}$.

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Q 2 Let us throw one dice three times, and let the number that comes up on the first throw be a, on the second throw be b, and on the third throw be c. Using these a, b and c, we consider the quadratic function $f(x) = ax^2 + bx + c$.

(1) The probability that $b = 4$ and that the quadratic equation $f(x) = 0$ has two different real solutions is $\dfrac{\boxed{\mathsf{N}}}{\boxed{\mathsf{OPQ}}}$.

(2) Let us find the probability that $f(10) > 453$.

The number of the cases of (a, b, c) such that $f(10) > 453$ is as follows:

when $a = 4$ and $b = 5$, it is $\boxed{\mathsf{R}}$;

when $a = 4$ and $b = 6$, it is $\boxed{\mathsf{S}}$;

when $a = 5$, it is $\boxed{\mathsf{TU}}$;

when $a = 6$, it is $\boxed{\mathsf{VW}}$.

Hence, the probability that $f(10) > 453$ is $\dfrac{\boxed{\mathsf{X}}}{\boxed{\mathsf{Y}}}$.

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This is the end of the questions for Part $\boxed{\text{I}}$. Leave the answer space $\boxed{\textbf{Z}}$ of Part $\boxed{\text{I}}$ blank.

II

Q 1 Let $x = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}$.

(1) We have $x = \boxed{\textsf{A}} + \sqrt{\boxed{\textsf{B}}}$ and $y = \boxed{\textsf{C}} - \sqrt{\boxed{\textsf{D}}}$. Hence we have

$$x + y = \boxed{\textsf{E}}, \quad xy = \boxed{\textsf{F}}, \quad \frac{1}{x^2} + \frac{1}{y^2} = \boxed{\textsf{GH}}.$$

Also we have

$$5(x^2 - 4x) + 3(y^2 - 4y + 1) = \boxed{\textsf{IJ}}.$$

(2) The values of the integers m and n such that $\dfrac{m}{x} + \dfrac{n}{y} = 4 + 4\sqrt{3}$ are

$$m = \boxed{\textsf{KL}}, \quad n = \boxed{\textsf{M}}.$$

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Q 2 Let us consider the three quadratic functions

$$f(x) = -x^2 - 2x + 1, \quad g(x) = -x^2 + 4x, \quad h(x) = 2x^2 + ax + b.$$

(1) When we denote the discriminant of the quadratic equation $h(x) - f(x) = 0$ by D_1 and the discriminant of the quadratic equation $h(x) - g(x) = 0$ by D_2, we have

$$D_1 = \boxed{\mathbf{N}}, \quad D_2 = \boxed{\mathbf{O}}$$

(for $\boxed{\text{N}}$ and $\boxed{\text{O}}$, choose the correct answers from among choices ⓪ ～ ⑤ below).

⓪ $a^2 + 4a - 3b + 7$ ① $a^2 - 8a - 12b + 16$ ② $a^2 + 4a - 12b + 16$

③ $a^2 + 8a + 12b + 16$ ④ $a^2 - 4a + 12b + 16$ ⑤ $a^2 - 8a - 3b + 7$

(2) The values of a and b such that both of the two equations $f(x) = h(x)$ and $g(x) = h(x)$ have only one real solution are

$$a = \boxed{\mathbf{P}}, \quad b = \frac{\boxed{\mathbf{Q}}}{\boxed{\mathbf{R}}}.$$

In this case, the solution of $f(x) = h(x)$ is $x = -\dfrac{\boxed{\mathbf{S}}}{\boxed{\mathbf{T}}}$ and the solution of $g(x) = h(x)$ is $x = \dfrac{\boxed{\mathbf{U}}}{\boxed{\mathbf{V}}}$.

(3) Let $b = 3$. Then the range of the values of a such that both $f(x) < h(x)$ and $g(x) < h(x)$ hold for any x is $\boxed{\mathbf{W}}$ (for $\boxed{\text{W}}$, choose the correct answer from among choices ⓪ ～ ⑤ below).

⓪ $-2 - 2\sqrt{6} < a < 10$ ① $a < -2 - 2\sqrt{6},\ 10 < a$

② $a < -1 - \sqrt{6},\ 10 < a$ ③ $-2 < a < -1 + \sqrt{6}$

④ $-2 < a < -2 + 2\sqrt{6}$ ⑤ $-1 - \sqrt{6} < a < 10$

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This is the end of the questions for Part $\boxed{\text{II}}$. Leave the answer spaces $\boxed{\textbf{X}}$ ～ $\boxed{\textbf{Z}}$ of Part $\boxed{\text{II}}$ blank.

III

Answer the following questions.

(1) The prime factorization of 1400 is

$$1400 = \boxed{\mathsf{A}}^{\boxed{\mathsf{B}}} \times \boxed{\mathsf{C}}^{\boxed{\mathsf{D}}} \times \boxed{\mathsf{E}}$$

(give the answers in the order $\boxed{\mathsf{A}} < \boxed{\mathsf{C}}$).

(2) The number of the divisors of 1400 is $\boxed{\mathsf{FG}}$.

(3) Let a and b be any two divisors of 1400 satisfying $1 < a < b$. There are $\boxed{\mathsf{H}}$ pairs (a, b) such that a and b are relatively prime and $ab = 1400$. Among them, a and b such that $b - a$ is maximized are

$$a = \boxed{\mathsf{I}}, \quad b = \boxed{\mathsf{JKL}}.$$

(4) For $a = \boxed{\mathsf{I}}$ and $b = \boxed{\mathsf{JKL}}$, consider the equation

$$bx - ay = 1. \qquad \cdots\cdots\cdots \quad ①$$

We can transform ① into the following equation:

$$y = \boxed{\mathsf{MN}}\,x + \frac{\boxed{\mathsf{O}}\,x - \boxed{\mathsf{P}}}{\boxed{\mathsf{Q}}}.$$

Therefore, among the pairs of positive integers x and y that satisfy equation ①, the pair such that x is minimized is

$$x = \boxed{\mathsf{R}}, \quad y = \boxed{\mathsf{ST}}.$$

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This is the end of the questions for Part $\boxed{\text{III}}$. Leave the answer spaces $\boxed{\textbf{U}}$ ～ $\boxed{\textbf{Z}}$ of Part $\boxed{\text{III}}$ blank.

IV

Let the quadrangle ABCD be a rhombus where the length of the sides is $\sqrt{2}$ and $\angle \mathrm{ABC} = 30^\circ$.

(1) We have

$$\mathrm{AC}^2 = \boxed{\mathbf{A}} - \boxed{\mathbf{B}}\sqrt{\boxed{\mathbf{C}}}, \quad \mathrm{BD}^2 = \boxed{\mathbf{D}} + \boxed{\mathbf{E}}\sqrt{\boxed{\mathbf{F}}}.$$

Now, for any positive numbers a and b, we have

$$\left(\sqrt{a} \pm \sqrt{b}\,\right)^2 = a + b \pm 2\sqrt{ab} \quad \text{(double-sign correspondence).}$$

Using this formula, we obtain

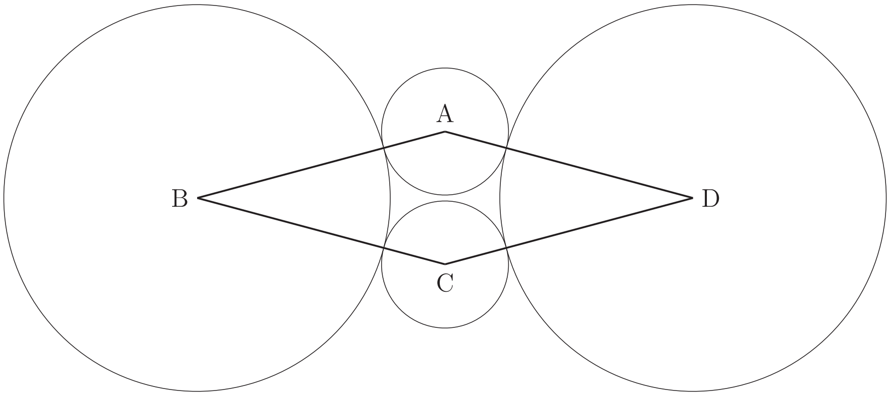
$$\mathrm{AC} = \sqrt{\boxed{\mathbf{G}}} - \boxed{\mathbf{H}}, \quad \mathrm{BD} = \sqrt{\boxed{\mathbf{I}}} + \boxed{\mathbf{J}}.$$

(This question is continued on the next page.)

(2) Let us draw four circles, each centered on one vertex of rhombus ABCD, with the following conditions:

The radii of the circles centered on vertices A and C are of length r, and those centered on vertices B and D are of length $\sqrt{2}-r$.

Circles centered on opposite vertices (A and C, B and D) may touch each other but may not intersect.



Let us denote the area of the region common to rhombus ABCD and these four circles by S. We have

$$S = \pi\left(r^2 - \frac{\sqrt{\boxed{\mathbf{K}}}}{\boxed{\mathbf{L}}}r + \frac{\boxed{\mathbf{M}}}{\boxed{\mathbf{N}}}\right),$$

where the range of r is

$$\sqrt{\boxed{\mathbf{O}}} - \frac{\sqrt{\boxed{\mathbf{P}}}+\boxed{\mathbf{Q}}}{\boxed{\mathbf{R}}} \leqq r \leqq \frac{\sqrt{\boxed{\mathbf{S}}}-\boxed{\mathbf{T}}}{\boxed{\mathbf{U}}}.$$

Hence S is minimized when $r = \dfrac{\sqrt{\boxed{\mathbf{V}}}}{\boxed{\mathbf{W}}}$, and the value of S then is $\dfrac{\boxed{\mathbf{X}}}{\boxed{\mathbf{YZ}}}\pi$.

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This is the end of the questions for Part $\boxed{\text{IV}}$.

This is the end of the questions for Course 1. Leave the answer spaces for Part $\boxed{\text{V}}$ blank.

Please check once more that you have properly marked your course number as "Course 1" on your answer sheet.

Do not take this question booklet out of the room.

Mathematics Course 2

(Advanced Course)

Marking Your Choice of Course on the Answer Sheet

Choose to answer <u>either</u> Course 1 or Course 2.

If you choose Course 2, for example, circle the label “Course 2” and completely fill in the oval under the label on your answer sheet as shown in the example on the right.

< Example >

解答コース Course	
コース 1 Course 1	コース 2 Course 2
○	●

<u>**If you do not correctly fill in the appropriate oval, your answers will not be graded.**</u>

I

Q 1 Let us consider the maximum value M and the minimum value m of the quadratic function

$$f(x) = x^2 - 2(a+1)x + 2a^2$$

over $0 \leqq x \leqq 2$, where a is a constant and $0 \leqq a \leqq 3$.

(1) The coordinates of the vertex of the graph of $y = f(x)$ are

$$\left(a + \boxed{\mathbf{A}},\ a^2 - \boxed{\mathbf{B}}\,a - \boxed{\mathbf{C}}\right).$$

(2) For $\boxed{\mathbf{D}}$ ~ $\boxed{\mathbf{H}}$ in the following sentences, choose the correct answers from among choices ⓪ ~ ⑨ below.

Let us find the maximum value M and the minimum value m according to the position of the axis of symmetry. We have that

if $0 \leqq a < \boxed{\mathbf{D}}$, then

$$M = \boxed{\mathbf{E}}, \quad m = \boxed{\mathbf{F}};$$

if $\boxed{\mathrm{D}} \leqq a \leqq 3$, then

$$M = \boxed{\mathbf{G}}, \quad m = \boxed{\mathbf{H}}.$$

⓪ 0	① 1	② 2	③ 3
④ $a^2 - 2a$	⑤ $a^2 - 2a - 1$	⑥ $2a^2$	
⑦ $2a^2 - 2a - 1$	⑧ $2a^2 - 4a$	⑨ $2a^2 - 6a + 3$	

(3) Thus, m is maximized at $a = \boxed{\mathbf{I}}$ and the value of m then is $\boxed{\mathbf{J}}$. Also, m is minimized at $a = \boxed{\mathbf{K}}$ and the value of m then is $\boxed{\mathbf{LM}}$.

- memo -

Q 2 Let us throw one dice three times, and let the number that comes up on the first throw be a, on the second throw be b, and on the third throw be c. Using these a, b and c, we consider the quadratic function $f(x) = ax^2 + bx + c$.

(1) The probability that $b = 4$ and that the quadratic equation $f(x) = 0$ has two different real solutions is $\dfrac{\boxed{\mathsf{N}}}{\boxed{\mathsf{OPQ}}}$.

(2) Let us find the probability that $f(10) > 453$.

The number of the cases of (a, b, c) such that $f(10) > 453$ is as follows:

when $a = 4$ and $b = 5$, it is $\boxed{\mathsf{R}}$;

when $a = 4$ and $b = 6$, it is $\boxed{\mathsf{S}}$;

when $a = 5$, it is $\boxed{\mathsf{TU}}$;

when $a = 6$, it is $\boxed{\mathsf{VW}}$.

Hence, the probability that $f(10) > 453$ is $\dfrac{\boxed{\mathsf{X}}}{\boxed{\mathsf{Y}}}$.

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This is the end of the questions for Part $\boxed{\text{I}}$. Leave the answer space $\boxed{\textbf{Z}}$ of Part $\boxed{\text{I}}$ blank.

II

Q 1 The sequence $\{a_n\}$ is defined by

$$a_1 = \frac{2}{9}, \quad a_n = \frac{(n+1)(2n-3)}{3n(2n+1)}\,a_{n-1} \quad (n = 2, 3, 4, \cdots).$$

We are to find the general term a_n and the infinite sum $\displaystyle\sum_{n=1}^{\infty} a_n$.

(1) For $\boxed{\textsf{A}}$ ～ $\boxed{\textsf{E}}$ in the following sentences, choose the correct answers from among ⓪ ～ ⑨ below.

First, when we set $b_n = \dfrac{n+1}{3^n a_n}$ and express $\dfrac{b_n}{b_{n-1}}$ in terms of n, we have

$$\frac{b_n}{b_{n-1}} = \frac{\boxed{\textsf{A}}}{\boxed{\textsf{B}}} \cdot \frac{a_{n-1}}{a_n} = \frac{\boxed{\textsf{C}}}{\boxed{\textsf{D}}}.$$

From this equation, we have

$$a_n = \frac{n+1}{3^n(\boxed{\textsf{E}})(2n+1)}.$$

⓪ $n-1$　① n　② $n+1$　③ $2n-1$　④ $2n+1$

⑤ $2n-3$　⑥ $2n+3$　⑦ $3n-1$　⑧ $3n$　⑨ $3n+1$

(2) Next, let $c_n = \dfrac{1}{3^n(2n+1)}$ $(n = 0, 1, 2, \cdots)$. When we set $a_n = Ac_{n-1} + Bc_n$, we see that $A = \dfrac{\boxed{\textsf{F}}}{\boxed{\textsf{G}}}$ and $B = \dfrac{\boxed{\textsf{HI}}}{\boxed{\textsf{J}}}$. Using this result to find $S_n = \displaystyle\sum_{k=1}^{n} a_k$, we have

$$S_n = \frac{\boxed{\textsf{K}}}{\boxed{\textsf{L}}}(\boxed{\textsf{M}} - c_n).$$

Hence we obtain

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n = \frac{\boxed{\textsf{N}}}{\boxed{\textsf{O}}}.$$

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Q 2 Let C be a circle with a radius of 4, centered at the point $(5, 0)$ on the x-axis.

(1) If $\mathrm{P}(p, q)$ is a point on circle C, then

$$p^2 - \boxed{\mathbf{PQ}}\,p + q^2 + \boxed{\mathbf{R}} = 0.$$

Also, the equation of the tangent to circle C at point $\mathrm{P}(p, q)$ is

$$(p - \boxed{\mathbf{S}})x + qy = \boxed{\mathbf{T}}\,p - \boxed{\mathbf{U}}.$$

(2) Let us draw a line tangent to circle C from point $\mathrm{A}(0, a)$ on the y-axis, where $a \geqq 0$, and let $\mathrm{P}(p, q)$ be the tangent point.

The length of the segment AP is minimized at $a = \boxed{\mathbf{V}}$, and the length in this case is $\boxed{\mathbf{W}}$.

Furthermore, the two tangents to circle C from point A are orthogonal when the length of AP is $\boxed{\mathbf{X}}$. In this case, the value of a is $a = \sqrt{\boxed{\mathbf{Y}}}$.

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This is the end of the questions for Part $\boxed{\text{II}}$. Leave the answer space $\boxed{\textbf{Z}}$ of Part $\boxed{\text{II}}$ blank.

III

Given the function

$$f(x) = x^3 - 3ax^2 - 3(2a+1)x + a + 2,$$

answer the following questions.

(1) For $\boxed{\mathbf{G}}$ ～ $\boxed{\mathbf{K}}$, choose the correct answers from among ⓪ ～ ⑤ below, and for the other $\boxed{}$, enter the correct numbers.

Since

$$f'(x) = \boxed{\mathbf{A}}\left(x - \boxed{\mathbf{B}}\,a - \boxed{\mathbf{C}}\right)\left(x + \boxed{\mathbf{D}}\right),$$

we see that

(i) when $a > \boxed{\mathbf{EF}}$, $f(x)$ is $\boxed{\mathbf{G}}$ at $x = -\boxed{\mathrm{D}}$ and is $\boxed{\mathbf{H}}$ at $x = \boxed{\mathrm{B}}\,a + \boxed{\mathrm{C}}$;

(ii) when $a = \boxed{\mathrm{EF}}$, $f(x)$ is always $\boxed{\mathbf{I}}$;

(iii) when $a < \boxed{\mathrm{EF}}$, $f(x)$ is $\boxed{\mathbf{J}}$ at $x = -\boxed{\mathrm{D}}$ and is $\boxed{\mathbf{K}}$ at $x = \boxed{\mathrm{B}}\,a + \boxed{\mathrm{C}}$.

⓪ locally maximized	① locally minimized	② increasing
③ decreasing	④ maximized	⑤ minimized

(This question is continued on the next page.)

(2) When we express the minimum value m of $f(x)$ over the range $-1 \leqq x \leqq 1$ in terms of a, we have that

(i) when $a \geqq \boxed{\mathsf{L}}$, $m = \boxed{\mathsf{MN}}\,a$;

(ii) when $\boxed{\mathsf{OP}} \leqq a < \boxed{\mathsf{L}}$, $m = \boxed{\mathsf{QR}}\left(a^3 + \boxed{\mathsf{S}}\,a^2 + \boxed{\mathsf{T}}\,a\right)$;

(iii) when $a < \boxed{\mathsf{OP}}$, $m = \boxed{\mathsf{U}}\,a + \boxed{\mathsf{V}}$.

(3) The value of m in (2) is maximized at $a = \dfrac{-\boxed{\mathsf{W}} + \sqrt{\boxed{\mathsf{X}}}}{\boxed{\mathsf{Y}}}$.

This is the end of the questions for Part $\boxed{\text{III}}$. Leave the answer space $\boxed{\mathsf{Z}}$ of Part $\boxed{\text{III}}$ blank.

IV

Consider the two functions

$$y = x \log ax, \quad \cdots\cdots\cdots \quad ①$$

$$y = 2x - 3, \quad \cdots\cdots\cdots \quad ②$$

where $a > 0$, and where log is the natural logarithm.

(1) Let us find a such that the graph of ① is tangent to the graph of ②.

The equation of the tangent to the graph of ① at the point $(t,\ t \log at)$ is $\boxed{\mathbf{A}}$ (for $\boxed{\text{A}}$, choose the correct answer from among choices ⓪ ～ ③ below).

⓪ $y = (\log at + 1)x - t$ ① $y = (\log at + a)x - t$

② $y = (a \log t + 1)x + t$ ③ $y = (a \log t + a)x + t$

Hence, the graph of ① is tangent to the graph of ② when $a = \dfrac{e}{\boxed{\mathbf{B}}}$, and the coordinates of the tangent point are $(\boxed{\mathbf{C}}, \boxed{\mathbf{D}})$.

(2) When $a = \dfrac{e}{\boxed{\text{B}}}$, function ① is minimized at $x = \boxed{\mathbf{E}}\,e^{-\boxed{\mathbf{F}}}$, and in this case the minimum value is $-\boxed{\mathbf{G}}\,e^{-\boxed{\mathbf{H}}}$.

(This question is continued on the next page.)

(3) When $a = \dfrac{e}{\boxed{\mathsf{B}}}$, let us find the area S of the region bounded by the graphs of ① and ② and the x-axis.

For the following indefinite integral, we have

$$\int x \log ax \, dx = \boxed{\mathsf{I}} + C, \quad \text{where } C \text{ is an integral constant}$$

(for $\boxed{\mathsf{I}}$, choose the correct answer from among ⓪ ～ ③ below).

⓪ $\dfrac{1}{2}x^2 \log ax - \dfrac{1}{2}x^2$ ① $2x^2 \log ax - 2x^2$

② $\dfrac{1}{2}x^2 \log ax - \dfrac{1}{4}x^2$ ③ $2x^2 \log ax - 4x^2$

Hence we obtain

$$S = \frac{\boxed{\mathsf{J}}}{\boxed{\mathsf{K}}} e^{-\boxed{\mathsf{L}}}.$$

This is the end of the questions for Part $\boxed{\text{IV}}$.

Leave the answer spaces $\boxed{\mathsf{M}}$ ～ $\boxed{\mathsf{Z}}$ of Part $\boxed{\text{IV}}$ blank.

This is the end of the questions for Course 2. Leave the answer spaces for Part $\boxed{\text{V}}$ blank.

Please check once more that you have properly marked your course number as "Course 2" on your answer sheet.

Do not take this question booklet out of the room.