2019 Examination for Japanese University Admission for International Students

Mathematics (80 min.)

【Course 1 (Basic), Course 2 (Advanced)】

※ Choose one of these courses and answer its questions only.

I Rules of Examination

1. Do not leave the room without proctor's permission.
2. Do not take this question booklet out of the room.

II Instructions for the Question Booklet

1. Do not open this question booklet until instructed.
2. After being instructed, write your name and examination registration number in space provided below, as printed on your examination voucher.
3. Course 1 is on pages 1-13, and Course 2 is on pages 15-27.
4. If your question booklet is missing any pages, raise your hand.
5. You may write notes and calculations in the question booklet.

III Instructions for how to answer the questions

1. You must mark your answers on the answer sheet with an HB pencil.
2. Each letter **A**, **B**, **C**, $\cdots$ in the questions represents a numeral (from 0 to 9) or the minus sign ($-$). When you mark your answers, fill in the oval completely for each letter in the corresponding row of the answer sheet (mark-sheet).
3. Sometimes an answer such as $\boxed{\text{A}}$ or $\boxed{\text{BC}}$ is used later in the question. In such a case, the symbol is shaded when it is used later, as $\boxed{\text{A}}$ or $\boxed{\text{BC}}$.

   Note the following :

   (1) Reduce square roots ($\sqrt{}$) as much as possible.
   (Example: Express $\sqrt{32}$ as $4\sqrt{2}$, not as $2\sqrt{8}$ or $\sqrt{32}$.)

   (2) For fractions, attach the minus sign to the numerator, and reduce the fraction to its lowest terms.
   (Example: Substitute $\frac{1}{3}$ for $\frac{2}{6}$. Also simplify as follows:
   $-\frac{2}{\sqrt{6}} = \frac{-2\sqrt{6}}{6} = \frac{-\sqrt{6}}{3}$. Then apply $\frac{-\sqrt{6}}{3}$ to the answer.)

   (3) If your answer to $\dfrac{\boxed{\text{A}}\sqrt{\boxed{\text{B}}}}{\boxed{\text{C}}}$ is $\dfrac{-\sqrt{3}}{4}$, mark as shown below.

   (4) If the answer to $\boxed{\text{DE}}\,x$ is $-x$, mark "$-$" for **D** and "1" for **E** as shown below.

A	●	0	1	2	3	4	5	6	7	8	9
B	−	0	1	2	●	4	5	6	7	8	9
C	−	0	1	2	3	●	5	6	7	8	9
D	●	0	1	2	3	4	5	6	7	8	9
E	−	0	●	2	3	4	5	6	7	8	9

4. Carefully read the instructions on the answer sheet, too.

※ Once you are instructed to start the examination, fill in your examination registration number and name.

Examination registration number			*					*						
Name														

Mathematics Course 1

(Basic Course)

(Course 2 begins on page 15)

Marking Your Choice of Course on the Answer Sheet

Choose to answer <u>either</u> Course 1 or Course 2.

If you choose Course 1, for example, circle the label “Course 1” and completely fill in the oval under the label on your answer sheet as shown in the example on the right.

< Example >

解答コース Course	
コース 1 Course 1	コース 2 Course 2
●	○

<u>If you do not correctly fill in the appropriate oval, your answers will not be graded.</u>

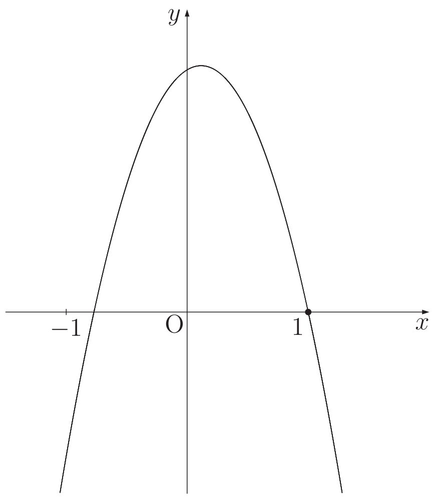
I

Q 1 For $\boxed{\textbf{A}}$ ～ $\boxed{\textbf{K}}$ in the following sentences, choose the correct answer from among choices ⓪ ～ ⑨ below.

(1) Consider the quadratic function

$$y = ax^2 + bx + c$$

whose graph is as shown in the figure at the right.



Then a, b and c satisfy the following expressions:

(i) a $\boxed{\textbf{A}}$ 0, b $\boxed{\textbf{B}}$ 0, c $\boxed{\textbf{C}}$ 0;

(ii) $a+b+c$ $\boxed{\textbf{D}}$ 0;

(iii) $a-b+c$ $\boxed{\textbf{E}}$ 0;

(iv) $4a+2b+c$ $\boxed{\textbf{F}}$ 0;

(v) b^2-4ac $\boxed{\textbf{G}}$ 0.

(2) Given the condition that a, b and c satisfy (i) and (ii) in (1), consider the case where the value of $a^2-8b-8c$ is minimized.

We see that $a =$ $\boxed{\textbf{H}}$. When we express $y = ax^2+bx+c$ in terms of b, we have

$$y = \boxed{\text{H}}\, x^2 + bx - b + \boxed{\textbf{I}}.$$

Also, we see that the range of the values of b is $\boxed{\textbf{J}} < b < \boxed{\textbf{K}}$.

⓪ 0	① 1	② 2	③ 3	④ 4
⑤ −2	⑥ −4	⑦ >	⑧ =	⑨ <

\- memo -

Q 2 Consider dice-X with the following property: when it is rolled, for numbers 1 through 5 the probabilities of that number's coming up are all the same, but the probability that 6 comes up is twice that of any other number.

(1) Denote by p the probability that a particular number 1 through 5 comes up. The probability that number 6 comes up is $\boxed{\mathbf{L}}\,p$. Since the probability of the whole event is $\boxed{\mathbf{M}}$, we have $p = \dfrac{\boxed{\mathbf{N}}}{\boxed{\mathbf{O}}}$.

(2) Dice-X is rolled twice in succession. Let us denote by A the event that for both rolls the number that comes up is 1 through 5, and by B the event that number 6 comes up at least once. Then the probability of event A, $P(A)$, and that of event B, $P(B)$, are

$$P(A) = \frac{\boxed{\mathbf{PQ}}}{\boxed{\mathbf{RS}}}, \quad P(B) = \frac{\boxed{\mathbf{TU}}}{\boxed{\mathbf{VW}}}.$$

Hence, we see that $\boxed{\mathbf{X}}$. (For $\boxed{\mathrm{X}}$, choose the correct answer from among choices ⓪ ～ ④ below.)

⓪ $P(A)$ is less than $P(B)$ and the difference between them is not less than $\dfrac{1}{36}$.

① $P(A)$ is less than $P(B)$ and the difference between them is less than $\dfrac{1}{36}$.

② $P(A)$ and $P(B)$ are the same.

③ $P(A)$ is greater than $P(B)$ and the difference between them is not less than $\dfrac{1}{36}$.

④ $P(A)$ is greater than $P(B)$ and the difference between them is less than $\dfrac{1}{36}$.

(This question is continued on the next page.)

(3) Next, dice-X is rolled three times in succession. Let us denote by C the event that for all three rolls the number which comes up is 1 through 5, and by D the event that the number 6 comes up at least once. When the probability $P(C)$ is compared with the probability $P(D)$, we see that $\boxed{\mathbf{Y}}$. (For $\boxed{\mathrm{Y}}$, choose the correct answer from among choices ⓪ ～ ④ below.)

⓪ $P(C)$ is less than $P(D)$ and $P(D)$ is not less than twice $P(C)$.

① $P(C)$ is less than $P(D)$ and $P(D)$ is less than twice $P(C)$.

② $P(C)$ and $P(D)$ are the same.

③ $P(C)$ is greater than $P(D)$ and $P(C)$ is not less than twice $P(D)$.

④ $P(C)$ is greater than $P(D)$ and $P(C)$ is less than twice $P(D)$.

This is the end of the questions for Part $\boxed{\mathrm{I}}$. Leave the answer space $\boxed{\mathbf{Z}}$ of Part $\boxed{\mathrm{I}}$ blank.

II

Q 1 Let $a = \sqrt{5} + \sqrt{3}$ and $b = \sqrt{5} - \sqrt{3}$. We are to find the integers x satisfying

$$2\left|x - \frac{a}{b}\right| + x < 10.$$

(1) We see that $\dfrac{a}{b} = \boxed{\text{A}} + \sqrt{\boxed{\text{BC}}}$. Hence the largest integer less than $\dfrac{a}{b}$ is $\boxed{\text{D}}$.

(2) For $\boxed{\text{F}}$ and $\boxed{\text{H}}$ in the following sentence, choose the correct answer from among choices ⓪ ～ ⑦ below, and for $\boxed{\text{E}}$ and $\boxed{\text{G}}$, enter the correct numbers.

When x is an integer, the left side of the inequality can be expressed without using the absolute value symbol as follows:

$$\begin{cases} \text{if } \ x \leqq \boxed{\text{E}}, \ \text{ then } \ 2\left|x - \dfrac{a}{b}\right| + x = \boxed{\text{F}}; \\ \text{if } \ x \geqq \boxed{\text{G}}, \ \text{ then } \ 2\left|x - \dfrac{a}{b}\right| + x = \boxed{\text{H}}. \end{cases}$$

⓪ $x - 6 - 2\sqrt{10}$ ① $x + 8 + 2\sqrt{15}$ ② $-x + 8 + 2\sqrt{15}$ ③ $-x + 6 + 2\sqrt{10}$

④ $3x - 6 - 2\sqrt{10}$ ⑤ $3x - 8 - 2\sqrt{15}$ ⑥ $-3x + 8 + 2\sqrt{15}$ ⑦ $-3x + 6 + 2\sqrt{10}$

(3) Thus, the integers x satisfying inequality $2\left|x - \dfrac{a}{b}\right| + x < 10$ are those greater than or equal to $\boxed{\text{I}}$ and less than or equal to $\boxed{\text{J}}$.

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Q 2 Let a be a real number. For the two quadratic functions in x

$$f(x) = x^2 + 2ax + a^2 - a,$$
$$g(x) = 4 - x^2,$$

answer the following questions.

(1) The range of the values of a such that the equation $f(x) = g(x)$ has two different solutions is

$$-\boxed{\mathbf{K}} < a < \boxed{\mathbf{L}}. \quad \cdots\cdots\cdots \quad ①$$

(2) In the case of (1), the parabolas $y = f(x)$ and $y = g(x)$ intersect at two points. We are to find the range of the values of a such that both of the y coordinates of these points of intersection are positive.

First, let $h(x) = f(x) - g(x)$. Since the solutions of the equation $f(x) = g(x)$ are the x coordinates of the points of intersection of parabolas $y = f(x)$ and $y = g(x)$, the solutions of $h(x) = 0$ have to be between $-\boxed{\mathbf{M}}$ and $\boxed{\mathbf{N}}$. Accordingly, we have

$$h(-\boxed{\mathrm{M}}) = a^2 - \boxed{\mathbf{O}}\,a + \boxed{\mathbf{P}} > 0, \quad \cdots\cdots\cdots \quad ②$$

$$h(\boxed{\mathrm{N}}) = a^2 + \boxed{\mathbf{Q}}\,a + \boxed{\mathbf{R}} > 0. \quad \cdots\cdots\cdots \quad ③$$

Also, from the position of the axis of the parabola $y = h(x)$ we have that

$$-\boxed{\mathbf{S}} < a < \boxed{\mathbf{T}}. \quad \cdots\cdots\cdots \quad ④$$

Therefore, from ①, ②, ③ and ④ we obtain

$$-\boxed{\mathbf{U}} < a < \boxed{\mathbf{V}}.$$

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This is the end of the questions for Part $\boxed{\text{II}}$. Leave the answer spaces $\boxed{\mathbf{W}}$ ～ $\boxed{\mathbf{Z}}$ of Part $\boxed{\text{II}}$ blank.

III

Where m and n are positive integers, consider the rational number

$$r = \frac{m}{3} + \frac{n}{7}.$$

We are to find m and n such that among all r's satisfying $r < \sqrt{2}$, r is closest to $\sqrt{2}$.

It is sufficient that among all m's and n's which satisfy the inequality

$$\boxed{\mathbf{A}}\,m + \boxed{\mathbf{B}}\,n < \boxed{\mathbf{CD}}\,\sqrt{2} \quad \cdots\cdots\cdots \quad ①$$

we find the m and n such that $\boxed{\text{A}}\,m + \boxed{\text{B}}\,n$ is closest to $\boxed{\text{CD}}\,\sqrt{2}$.

Squaring both sides of ①, we have

$$\left(\boxed{\text{A}}\,m + \boxed{\text{B}}\,n\right)^2 < \boxed{\mathbf{EFG}}.$$

Here, the greatest square number which is smaller than $\boxed{\text{EFG}}$ is $\boxed{\mathbf{HIJ}} = \boxed{\mathbf{KL}}^2$. So, let us consider the equation

$$\boxed{\text{A}}\,m + \boxed{\text{B}}\,n = \boxed{\text{KL}}.$$

Transforming this equation, we have

$$n = \frac{\boxed{\mathbf{MN}} - \boxed{\mathbf{O}}\,m}{\boxed{\mathbf{P}}}.$$

Since n is an integer, $\boxed{\text{MN}} - \boxed{\text{O}}\,m$ is a multiple of $\boxed{\mathbf{Q}}$.

Thus, we obtain

$$m = \boxed{\mathbf{R}}, \quad n = \boxed{\mathbf{S}}.$$

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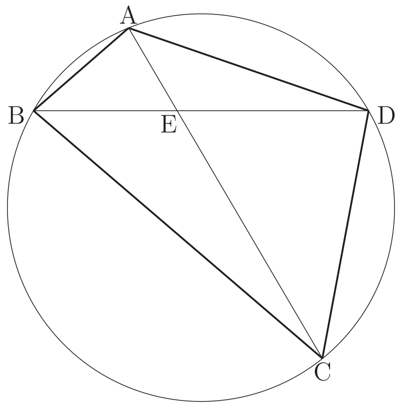
This is the end of the questions for Part $\boxed{\text{III}}$. Leave the answer spaces $\boxed{\textbf{T}}$ ～ $\boxed{\textbf{Z}}$ of Part $\boxed{\text{III}}$ blank.

IV

For a quadrilateral ABCD inscribed in a circle of radius 1, let AB : AD = 1 : 2 and ∠BAD = 120°. Also, when the point of intersection of diagonals BD and AC is denoted by E, let BE : ED = 3 : 4.

We are to find the area of quadrilateral ABCD.



In order to find the area of quadrilateral ABCD, we are to find the area of triangle ABD, denoted by △ABD, and the area of triangle BCD, denoted by △BCD.

First, let us find △ABD. Since

$$\mathrm{BD} = \sqrt{\boxed{\mathbf{A}}}, \quad \mathrm{AB} = \frac{\sqrt{\boxed{\mathbf{BC}}}}{\boxed{\mathbf{D}}},$$

we have

$$\triangle \mathrm{ABD} = \frac{\boxed{\mathbf{E}}\sqrt{\boxed{\mathbf{F}}}}{\boxed{\mathbf{GH}}}. \quad \cdots\cdots\cdots \quad ①$$

Next, let us find △BCD. Since

$$\triangle \mathrm{ABC} : \triangle \mathrm{ACD} = \boxed{\mathbf{I}} : \boxed{\mathbf{J}},$$

we see that BC : CD = $\boxed{\mathbf{K}} : \boxed{\mathbf{L}}$. (Give the answers using the simplest integer ratios.)

Hence we have $\mathrm{BC} = \dfrac{\boxed{\mathbf{M}}\sqrt{\boxed{\mathbf{NO}}}}{\boxed{\mathbf{P}}}$ and

$$\triangle \mathrm{BCD} = \frac{\boxed{\mathbf{Q}}\sqrt{\boxed{\mathbf{R}}}}{\boxed{\mathbf{ST}}}. \quad \cdots\cdots\cdots \quad ②$$

Thus, from ① and ② we obtain the result that the area of quadrilateral ABCD is $\dfrac{\boxed{\mathbf{U}}\sqrt{\boxed{\mathbf{V}}}}{\boxed{\mathbf{W}}}$.

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This is the end of the questions for Part $\boxed{\text{IV}}$.

Leave the answer spaces $\boxed{\textbf{X}}$ ~ $\boxed{\textbf{Z}}$ of Part $\boxed{\text{IV}}$ blank.

This is the end of the questions for Course 1. Leave the answer spaces for Part $\boxed{\text{V}}$ blank.

Please check once more that you have properly marked your course number as "Course 1" on your answer sheet.

Do not take this question booklet out of the room.

Mathematics Course 2

(Advanced Course)

Marking Your Choice of Course on the Answer Sheet

Choose to answer <u>either</u> Course 1 or Course 2.

If you choose Course 2, for example, circle the label “Course 2” and completely fill in the oval under the label on your answer sheet as shown in the example on the right.

< Example >

解答コース Course	
コース 1 Course 1	コース 2 Course 2
○	●

<u>**If you do not correctly fill in the appropriate oval, your answers will not be graded.**</u>

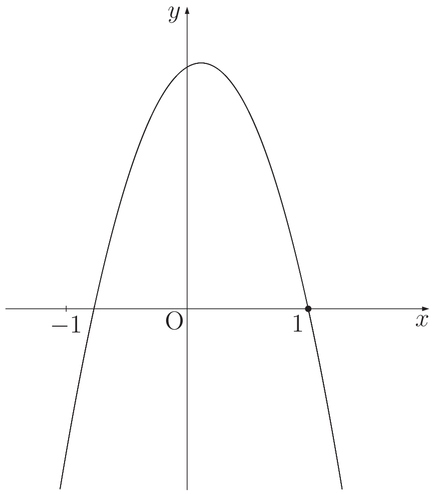
I

Q 1 For $\boxed{\textbf{A}}$ ～ $\boxed{\textbf{K}}$ in the following sentences, choose the correct answer from among choices ⓪ ～ ⑨ below.

(1) Consider the quadratic function

$$y = ax^2 + bx + c$$

whose graph is as shown in the figure at the right.



Then a, b and c satisfy the following expressions:

(i) a $\boxed{\textbf{A}}$ 0, b $\boxed{\textbf{B}}$ 0, c $\boxed{\textbf{C}}$ 0;

(ii) $a+b+c$ $\boxed{\textbf{D}}$ 0;

(iii) $a-b+c$ $\boxed{\textbf{E}}$ 0;

(iv) $4a+2b+c$ $\boxed{\textbf{F}}$ 0;

(v) b^2-4ac $\boxed{\textbf{G}}$ 0.

(2) Given the condition that a, b and c satisfy (i) and (ii) in (1), consider the case where the value of $a^2-8b-8c$ is minimized.

We see that $a = \boxed{\textbf{H}}$. When we express $y = ax^2+bx+c$ in terms of b, we have

$$y = \boxed{\text{H}}\,x^2 + bx - b + \boxed{\textbf{I}}.$$

Also, we see that the range of the values of b is $\boxed{\textbf{J}} < b < \boxed{\textbf{K}}$.

⓪ 0	① 1	② 2	③ 3	④ 4
⑤ −2	⑥ −4	⑦ >	⑧ =	⑨ <

- memo -

Q 2 Consider dice-X with the following property: when it is rolled, for numbers 1 through 5 the probabilities of that number's coming up are all the same, but the probability that 6 comes up is twice that of any other number.

(1) Denote by p the probability that a particular number 1 through 5 comes up. The probability that number 6 comes up is $\boxed{\mathbf{L}}\,p$. Since the probability of the whole event is $\boxed{\mathbf{M}}$, we have $p = \dfrac{\boxed{\mathbf{N}}}{\boxed{\mathbf{O}}}$.

(2) Dice-X is rolled twice in succession. Let us denote by A the event that for both rolls the number that comes up is 1 through 5, and by B the event that number 6 comes up at least once. Then the probability of event A, $P(A)$, and that of event B, $P(B)$, are

$$P(A) = \frac{\boxed{\mathbf{PQ}}}{\boxed{\mathbf{RS}}}, \quad P(B) = \frac{\boxed{\mathbf{TU}}}{\boxed{\mathbf{VW}}}.$$

Hence, we see that $\boxed{\mathbf{X}}$. (For $\boxed{\mathrm{X}}$, choose the correct answer from among choices ⓪ ～ ④ below.)

⓪ $P(A)$ is less than $P(B)$ and the difference between them is not less than $\dfrac{1}{36}$.

① $P(A)$ is less than $P(B)$ and the difference between them is less than $\dfrac{1}{36}$.

② $P(A)$ and $P(B)$ are the same.

③ $P(A)$ is greater than $P(B)$ and the difference between them is not less than $\dfrac{1}{36}$.

④ $P(A)$ is greater than $P(B)$ and the difference between them is less than $\dfrac{1}{36}$.

(This question is continued on the next page.)

(3) Next, dice-X is rolled three times in succession. Let us denote by C the event that for all three rolls the number which comes up is 1 through 5, and by D the event that the number 6 comes up at least once. When the probability $P(C)$ is compared with the probability $P(D)$, we see that $\boxed{\mathbf{Y}}$. (For $\boxed{\mathrm{Y}}$, choose the correct answer from among choices ⓪ ~ ④ below.)

⓪ $P(C)$ is less than $P(D)$ and $P(D)$ is not less than twice $P(C)$.

① $P(C)$ is less than $P(D)$ and $P(D)$ is less than twice $P(C)$.

② $P(C)$ and $P(D)$ are the same.

③ $P(C)$ is greater than $P(D)$ and $P(C)$ is not less than twice $P(D)$.

④ $P(C)$ is greater than $P(D)$ and $P(C)$ is less than twice $P(D)$.

This is the end of the questions for Part $\boxed{\mathrm{I}}$. Leave the answer space $\boxed{\mathbf{Z}}$ of Part $\boxed{\mathrm{I}}$ blank.

II

Q 1 For $\boxed{\textbf{A}}$, $\boxed{\textbf{B}}$, $\boxed{\textbf{D}}$, $\boxed{\textbf{E}}$ and $\boxed{\textbf{G}}$ in the following sentences, choose the correct answer from among choices ⓪ ～ ⑨ below, and for the other $\boxed{}$, enter the correct number.

Given a sphere of radius 2 with the center at point O, we have a tetrahedron ABCD whose four vertices are on the sphere. Let AB $=$ BC $=$ CA $= 2$ and side BD be a diameter of the sphere.

Set $\overrightarrow{\mathrm{OA}} = \vec{a}$, $\overrightarrow{\mathrm{OB}} = \vec{b}$ and $\overrightarrow{\mathrm{OC}} = \vec{c}$.

(1) Let M and N denote the midpoints of segments DA and BC, respectively. Then we have

$$\overrightarrow{\mathrm{DA}} = \boxed{\textbf{A}}, \quad \overrightarrow{\mathrm{MN}} = \frac{\boxed{\textbf{B}}}{\boxed{\textbf{C}}} + \boxed{\textbf{D}}.$$

(2) When the midpoint of segment MN is denoted by P and the center of gravity of triangle BCD is denoted by G, we see that

$$\overrightarrow{\mathrm{OP}} = \frac{\boxed{\textbf{E}}}{\boxed{\textbf{F}}}, \quad \overrightarrow{\mathrm{OG}} = \frac{\boxed{\textbf{G}}}{\boxed{\textbf{H}}}, \quad \left|\overrightarrow{\mathrm{PG}}\right| = \frac{\sqrt{\boxed{\textbf{I}}}}{\boxed{\textbf{J}}}.$$

Also, since $\overrightarrow{\mathrm{AG}} = \dfrac{\boxed{\textbf{K}}}{\boxed{\textbf{L}}}\overrightarrow{\mathrm{AP}}$, we see that the three points A, P and G are on a straight line.

⓪ $\vec{a}$	① $\vec{b}$	② $\vec{c}$	③ $\vec{a} - \vec{b}$	④ $\vec{b} - \vec{c}$
⑤ $\vec{c} - \vec{a}$	⑥ $\vec{a} + \vec{b}$	⑦ $\vec{b} + \vec{c}$	⑧ $\vec{c} + \vec{a}$	⑨ $\vec{a} + \vec{b} + \vec{c}$

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Q 2 Let α, β and γ be three complex numbers representing three different points A, B and C on a complex plane. Also, α, β and γ satisfy

$$(\gamma-\alpha)^2+(\gamma-\alpha)(\beta-\alpha)+(\beta-\alpha)^2 = 0, \qquad \cdots\cdots\cdots \quad ①$$

$$|\beta-2\alpha+\gamma| = 3. \qquad \cdots\cdots\cdots \quad ②$$

We are to find the area of the triangle ABC.

Since from ①

$$\frac{\gamma-\alpha}{\beta-\alpha} = \frac{-\boxed{\mathbf{M}}\pm\sqrt{\boxed{\mathbf{N}}}\,i}{\boxed{\mathbf{O}}},$$

we have

$$\left|\frac{\gamma-\alpha}{\beta-\alpha}\right| = \boxed{\mathbf{P}}, \quad \arg\frac{\gamma-\alpha}{\beta-\alpha} = \pm\frac{\boxed{\mathbf{Q}}}{\boxed{\mathbf{R}}}\pi,$$

where $-\pi < \arg\dfrac{\gamma-\alpha}{\beta-\alpha} < \pi$. Also, since

$$\beta-2\alpha+\gamma = (\beta-\alpha)\cdot\frac{\boxed{\mathbf{S}}\pm\sqrt{\boxed{\mathbf{T}}}\,i}{\boxed{\mathbf{U}}},$$

we have from ② that

$$|\beta-\alpha| = \boxed{\mathbf{V}}.$$

Thus we obtain the result that the area of triangle ABC is $\dfrac{\boxed{\mathbf{W}}\sqrt{\boxed{\mathbf{X}}}}{\boxed{\mathbf{Y}}}$.

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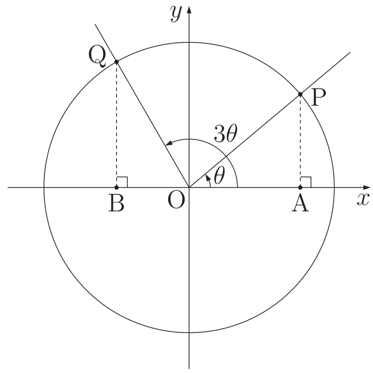
This is the end of the questions for Part $\boxed{\text{II}}$. Leave the answer space $\boxed{\textsf{Z}}$ of Part $\boxed{\text{II}}$ blank.

III

On a coordinate plane, consider a circle C with the radius of 1 centered at the origin O. We denote by P and Q the points of intersection of C and the radii which are rotated at angles of θ and 3θ respectively from the positive section of the x axis, where $0 \leqq \theta \leqq \pi$.



Also, we denote by A the point at which the straight line which is perpendicular to the x axis and passes through point P intersects the x axis, and we denote by B the point at which the straight line which is perpendicular to the x axis and passes through point Q intersects the x axis. Furthermore, we denote the length of line segment AB by ℓ.

(1) When $\theta = \dfrac{\pi}{3}$, we see that $\ell = \dfrac{\boxed{\mathbf{A}}}{\boxed{\mathbf{B}}}$.

(2) We are to find the maximum value of ℓ. When we set $\cos\theta = t$ and express ℓ in terms of t, we have

$$\ell = \left| \boxed{\mathbf{C}}\, t^{\boxed{\mathbf{D}}} - \boxed{\mathbf{E}}\, t \right|.$$

Next, when we set $g(t) = \boxed{\mathrm{C}}\, t^{\boxed{\mathrm{D}}} - \boxed{\mathrm{E}}\, t$, we have

$$g'(t) = \boxed{\mathbf{F}} \left(\boxed{\mathbf{G}}\, t^{\boxed{\mathbf{H}}} - 1 \right).$$

Hence, when

$$\cos\theta = \pm \frac{\sqrt{\boxed{\mathbf{I}}}}{\boxed{\mathbf{J}}},$$

ℓ is maximized and its value is $\dfrac{\boxed{\mathbf{K}}\sqrt{\boxed{\mathbf{L}}}}{\boxed{\mathbf{M}}}$.

(This question is continued on the next page.)

(3) For $\boxed{\mathbf{N}}$ ～ $\boxed{\mathbf{S}}$ in the following sentence, choose the correct answer from among choices ⓪ ～ ⑨ below.

There are two pairs of points P and Q at which ℓ is maximized, and their coordinates are

$$\mathrm{P}\left(\frac{\sqrt{\boxed{\mathbf{I}}}}{\boxed{\mathbf{J}}},\ \boxed{\mathbf{N}}\right),\quad \mathrm{Q}\left(\boxed{\mathbf{O}},\ \boxed{\mathbf{P}}\right)$$

and

$$\mathrm{P}\left(-\frac{\sqrt{\boxed{\mathbf{I}}}}{\boxed{\mathbf{J}}},\ \boxed{\mathbf{Q}}\right),\quad \mathrm{Q}\left(\boxed{\mathbf{R}},\ \boxed{\mathbf{S}}\right).$$

⓪ $\dfrac{\sqrt{6}}{3}$　① $\dfrac{\sqrt{6}}{2}$　② $\dfrac{4\sqrt{3}}{9}$　③ $-\dfrac{4\sqrt{3}}{9}$

④ $\dfrac{5\sqrt{3}}{9}$　⑤ $-\dfrac{5\sqrt{3}}{9}$　⑥ $\dfrac{\sqrt{6}}{9}$　⑦ $-\dfrac{\sqrt{6}}{9}$

⑧ $\dfrac{2\sqrt{6}}{9}$　⑨ $-\dfrac{2\sqrt{6}}{9}$

This is the end of the questions for Part III. Leave the answer spaces $\boxed{\mathbf{T}}$ ～ $\boxed{\mathbf{Z}}$ of Part III blank.

IV

Answer the following questions, where log is the natural logarithm.

(1) Let $f(x) = x - 1 - \log x$. We are to find the minimum value of $f(x)$.

First, we have

$$f'(x) = \boxed{\mathsf{A}} - \frac{\boxed{\mathsf{B}}}{x}.$$

Examining the increases and decreases of the value of $f(x)$, we see that at $x = \boxed{\mathsf{C}}$ the function is minimized and its value is $\boxed{\mathsf{D}}$. From this, we derive the inequality $x - 1 \geqq \log x$.

(2) For $\boxed{\mathsf{G}}$ in the following sentences, choose the correct answer from among choices ⓪ ~ ③ below. For the other $\boxed{}$, enter the correct number.

Let k be a positive real number and n be a positive integer. We denote by S the area of the figure bounded by the three straight lines $y = \dfrac{x}{k} - 1$, $x = n$, $x = n + 1$ and the curve $y = \log \dfrac{x}{k}$. We are to express S in terms of k and n.

Using the result of (1), we have

$$S = \left[\frac{x^{\boxed{\mathsf{E}}}}{\boxed{\mathsf{F}}\,k} - x - \boxed{\mathsf{G}} + x \log k \right]_n^{n+1}$$

$$= \frac{\boxed{\mathsf{H}}\,n + 1}{\boxed{\mathsf{I}}\,k} + \log k - (n + \boxed{\mathsf{J}}) \log(n + \boxed{\mathsf{K}}) + n \log n.$$

⓪ $x(\log x + 1)$ ① $x(\log x - 1)$ ② $\dfrac{\log x + 1}{x}$ ③ $\dfrac{\log x - 1}{x}$

(This question is continued on the next page.)

(3) When n is a fixed positive integer and k is moved within the range of $k > 0$, denote the minimum value of S in (2) by a_n. We are to find a_n and $\lim\limits_{n\to\infty} a_n$.

Differentiating S by k, we have

$$\frac{dS}{dk} = \frac{\boxed{\mathsf{L}}\,k - (\boxed{\mathsf{M}}\,n+1)}{\boxed{\mathsf{N}}\,k^2}.$$

Hence S is minimized at $k = n + \dfrac{\boxed{\mathsf{O}}}{\boxed{\mathsf{P}}}$. From this, we obtain

$$a_n = \boxed{\mathsf{Q}} - \log\left\{\left(\boxed{\mathsf{R}} + \frac{\boxed{\mathsf{S}}}{n}\right)^n \cdot \frac{\boxed{\mathsf{T}}\,n + \boxed{\mathsf{U}}}{\boxed{\mathsf{V}}\,n+1}\right\},$$

and so

$$\lim_{n\to\infty} a_n = \boxed{\mathsf{W}}.$$

This is the end of the questions for Part $\boxed{\text{IV}}$.

Leave the answer spaces $\boxed{\mathsf{X}}$ ~ $\boxed{\mathsf{Z}}$ of Part $\boxed{\text{IV}}$ blank.

This is the end of the questions for Course 2. Leave the answer spaces for Part $\boxed{\text{V}}$ blank.

Please check once more that you have properly marked your course number as "Course 2" on your answer sheet.

Do not take this question booklet out of the room.