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Estimating Cambodia's Economic Condition by Dynamic Factor Model

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Contents

I. Motivation of the Study

- Lack of early warning system to capture the recession.
- Monitoring economic activity, mainly to observe the business cycle, requires high-frequency indicator that could represent the cyclical movement of the economy.
- Coincident Index (CI) could observe the movement of economic condition or capture the state of economy.
- Research Questions

1. How is the current state of Cambodia's economic condition?
2. How does Cambodia's fiscal position comove with the state of the economy?

II. Literature review

- Under a Gaussian assumption on disturbance with known parameters, Kalman filter provided the best linear unbiased predictions of the common factors in the context of the linear state-space model (Rodríguez and Ruiz, 2012).
- Flexible in working with non-stationary datasets and strong correlation of idiosyncratic noises, imposing restrictions, and handling irregular elements and missing observation (Poncela & Ruiz, 2012).
- Forni, Hallin, Lippi, and Reichlin (2000) showed the asymptotic consistency of the estimated common factors of the DFM. Stock and Watson (2002), Bai and Ng (2002), and Bai (2003) derived the asymptotic consistency and normality of estimated common factors and factor loadings of the PC.

- For asymptotic property, the estimated factors by DFM and PC are consistent with cross-sectional (i) and time dimensions (T) (i; T$\rightarrow \infty$).
- However, later studies indicate that the cross-sectional dimension does not have to be large for a consistent estimation.
- Caggiano, Kapetanios, and Labhard (2011) showed that 12 to 22 variables could achieve the best result in extracting common factors.
- Doz, Giannone, and Reichlin (2011) estimated the unobserved common factors by the two-step procedure of combining PC and Kalman smoother.

- With the Gaussian assumption, the parameters are estimated by the Maximum Likelihood (ML).
- The asymptotic consistency the ($\underset{T\to\infty}{\text{plim}}(\hat{\varphi}) = \varphi$) and normality of the estimated parameters of the DFM were shown in Caines (1988 p. 426); Durbin and Koopman (2012).
- The EM algorithm is a common tool to estimate the parameters in the MLE. An alternative algorithm, Newton-Raphson, showed a faster convergence rate (Lisdfgyuiop ndstrom & Bates, 1988).
- However, in estimating many parameters, the Newton-Raphson algorithm could be unstable in the iteration process unless the initial guesses were close to the true values (Wilks, 2019, p. 128).

III. Methodology

- The study follows the state-space or dynamic factor model. Estimating the unobserved state follows the two-step procedure proposed by Doz, Giannone, and Reichlin (2011).

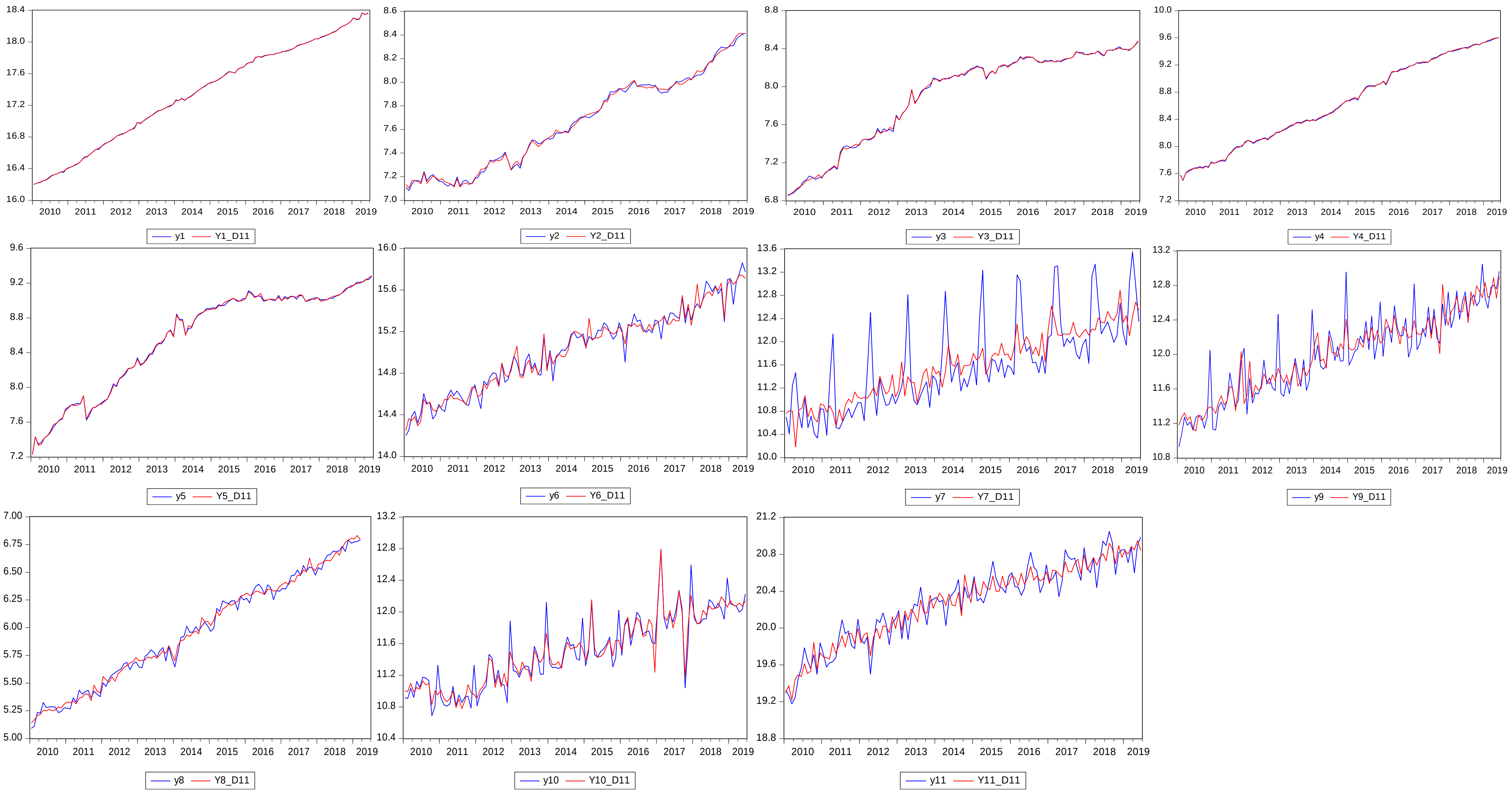
1. Data

- Total Bank credits (Y_1); Bank lending to the service-related sectors (Y_2); Bank lending to the manufacturing sector(Y_3); Bank lending to the retail trade sector (Y_4); Bank lending to the wholesale sector (Y_5); Total electricity supply (Y_6); Export Value (Y_7); Import Value (Y_8); Corporate income or profit tax (Y_9); Domestic VAT (Y_{10}); Import VAT (Y_{11})
- Official exchange rate (Z_1); Broad money supply (Z_2); Bank lending rate (Z_3)

2. Treatment of data

- Seasonally adjusted and normalized data
- Non-stationary data in extracting the common factor

Figure 1 Seasonal and non-seasonal adjusted of the series



3. Model (ARMAX linear state space)

- The ARMAX linear state-space model can be written as:

$$\text{observation equation } Y_t = \rho_t S_t + \alpha_t Z_t + u_t \ (1)$$

$$\text{transition equation } S_{t+1} = \theta_t S_t + v_t \ (2)$$

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} \sim (\text{i.i.d})\ N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} R_t & 0 \\ 0 & Q_t \end{bmatrix}\right)$$

- Y_t is $m \times 1$; S_t is $n \times 1$; Z_t is $p \times 1$
- ρ_t is $m \times n$, α_t is $m \times p$, and θ_t is $n \times n$ (where only $\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{23}, \alpha_{33}, \alpha_{43}, \alpha_{53}, \alpha_{71}$, and α_{81} are non-zero coefficients while the other parameters are restricted as zero).

3.1. State estimation

- The study follows Shumway and Stoffer (2017) textbook to derive this.
- Kalman Filter for state estimation

$$S_{t+1}^{t} = \theta_t S_t^t \ (3)$$

$$P_{t+1}^{t} = \theta_t P_t^t \theta_t' + Q_t (4)$$

$$S_{t+1}^{t+1} = S_{t+1}^{t} + K_{t+1}\epsilon_{t+1} \text{ ; where } [\epsilon_{t+1} = Y_{t+1} - (\rho_{t+1} S_{t+1}^{t} + \alpha_{t+1} Z_{t+1})] \ (5)$$

$$P_{t+1}^{t+1} = (I - K_{t+1}\rho_{t+1}) P_{t+1}^{t}\text{; where I is the identity matrix } (6)$$

$$\text{The Kalman gain } (K_{t+1})\text{: } K_{t+1} = P_{t+1}^{t}\rho_{t+1}'(\rho_{t+1} P_{t+1}^{t} \rho_{t+1}' + R_{t+1})^{-1} (7)$$

- Kalman Smoother for state estimation

$$S_t^n = S_t^t + J_t(S_{t+1}^n - S_{t+1}^t) \ (8)$$

$$P_t^n = P_t^t + J_t(P_{t+1}^n - P_{t+1}^t) J_t' \ (9)$$

$$\text{Where } J_t = P_t^t \theta_t' (P_{t+1}^t)^{-1} (10)$$

3.2. Parameters estimation

- It replaces parameters, ρ_t, in the system by their consistent estimators $\hat{\rho}_t$.
- Using the state variable generated by the PC, the study estimates the initial weights by the OLS and FGLS.
- Let $\varphi = \{S_0^0, P_0^0, Q_t, \theta_t, \alpha_t, R_t\}$ is the vector of the parameters to be estimated with the initial state $S_o \sim N(S_0^0, P_0^0)$. The likelihood is calculated from the innovations $\epsilon_1, \epsilon_2, \dots, \epsilon_t$.

$$\epsilon_t = Y_t - (\rho_t S_t^{t-1} + \alpha_t Z_t) \; ; \; \epsilon_t \sim N(0, \textstyle\sum_t) \quad (11)$$

- Ignoring the constant, the log-likelihood of $\log L(\varphi)$ is

$$\log L(\varphi) = -\frac{1}{2}\sum_{t=1}^{n} \log\left|\textstyle\sum_t(\varphi)\right| - \frac{1}{2}\sum_{t=1}^{n} \epsilon_t(\varphi)' \textstyle\sum_t(\varphi)^{-1} \epsilon_t(\varphi) \quad (12)$$

- Asymptotic property of consistency and normality of estimators hold in general (Shumway & Stoffer, 2017).

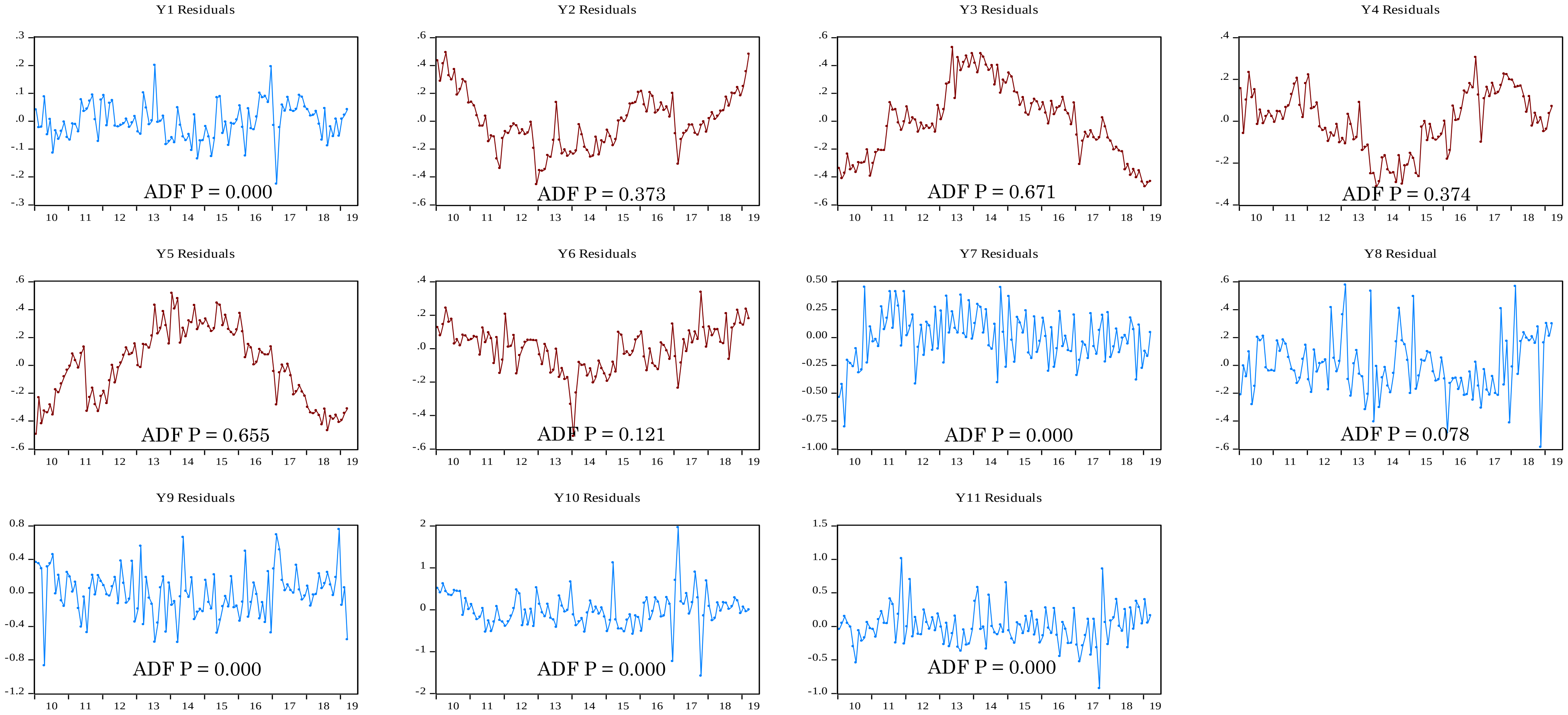
IV. Result of the study

Table 1. Descriptive statistics

Variables	Mean	SD	Unit Root test (Level)		Unit Root test (1st difference)	
			ADF	P-Value	ADF	P-Value
Total Bank credits (Y_1)	17.246	0.717	-0.410	0.903	-14.049	0.000
Bank lending to the service-related sectors (Y_2)	7.652	0.396	0.763	0.993	-12.542	0.000
Bank lending to the manufacturing sector (Y_3)	7.885	0.503	-3.017	0.057	-13.758	0.000
Bank lending to the retail trade sector (Y_4)	8.609	0.640	-0.633	0.858	-13.001	0.000
Bank lending to the wholesale sector (Y_5)	8.539	0.608	-2.312	0.170	-14.933	0.000
Total electricity supply (Y_6)	5.796	0.588	0.350	0.980	-12.247	0.000
Export Value (Y_7)	19.611	0.798	-0.781	0.822	-21.410	0.000
Import Value (Y_8)	14.252	0.961	-1.130	0.704	-9.796	0.000
Corporate income or profit tax (Y_9)	11.746	0.588	-0.194	0.935	-8.347	0.000
Domestic VAT (Y_{10})	11.521	0.432	-0.412	0.902	-8.870	0.000
Import VAT (Y_{11})	12.029	0.452	-0.299	0.920	-9.615	0.000

1. State estimation

Figure 2 Residuals of OLS regression of each series I(1) on the state variable I(1)



1.1. Dynamic factor model (ARMAX)

- ARMAX (4,2,0)
- Using the state variable generated by the PC, it constructs the state equation of the DFM in AR form.

Table 3 Lag selection criteria of the state variable

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-257.299	NA	8.825	5.016	5.041	5.026
1	-5.118	494.568	0.067	0.138	0.189	0.159
2	0.084	10.102	0.062	0.057	0.133	0.088
3	7.206	13.689*	0.055	-0.062	0.040*	-0.021
4	8.953	3.325	0.054*	-0.076*	0.051	-0.024*
5	8.970	0.032	0.055	-0.058	0.096	0.004

- Additionally, it replaces the parameters ρ_t with $\hat{\rho}_t$ estimated by the OLS and FGLS.

1.2. Dynamic factor model (ARMAX)

Figure 3 State estimation by Kalman smoother and Kalman filter

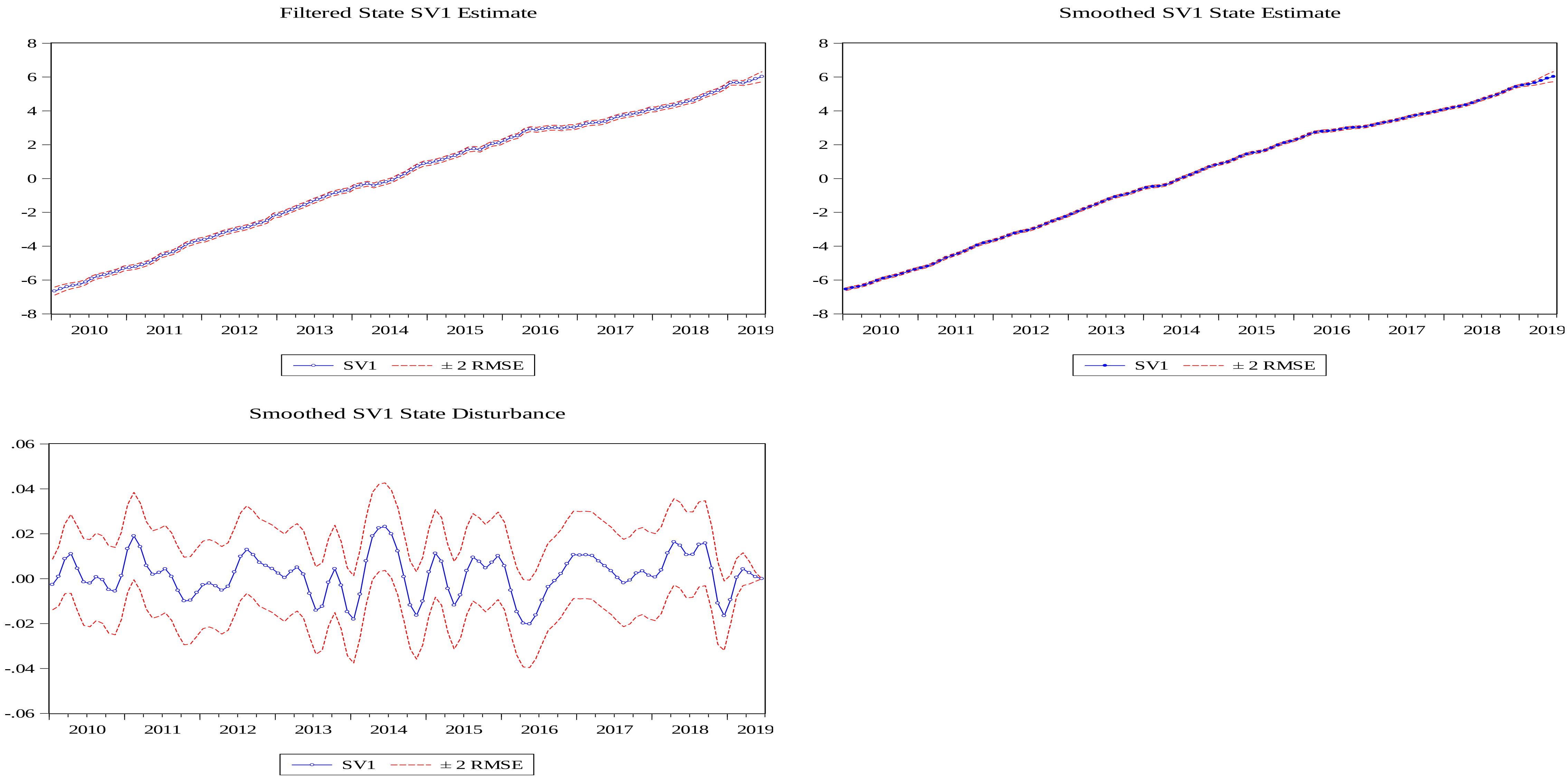


Figure 4 Residual diagnostics check

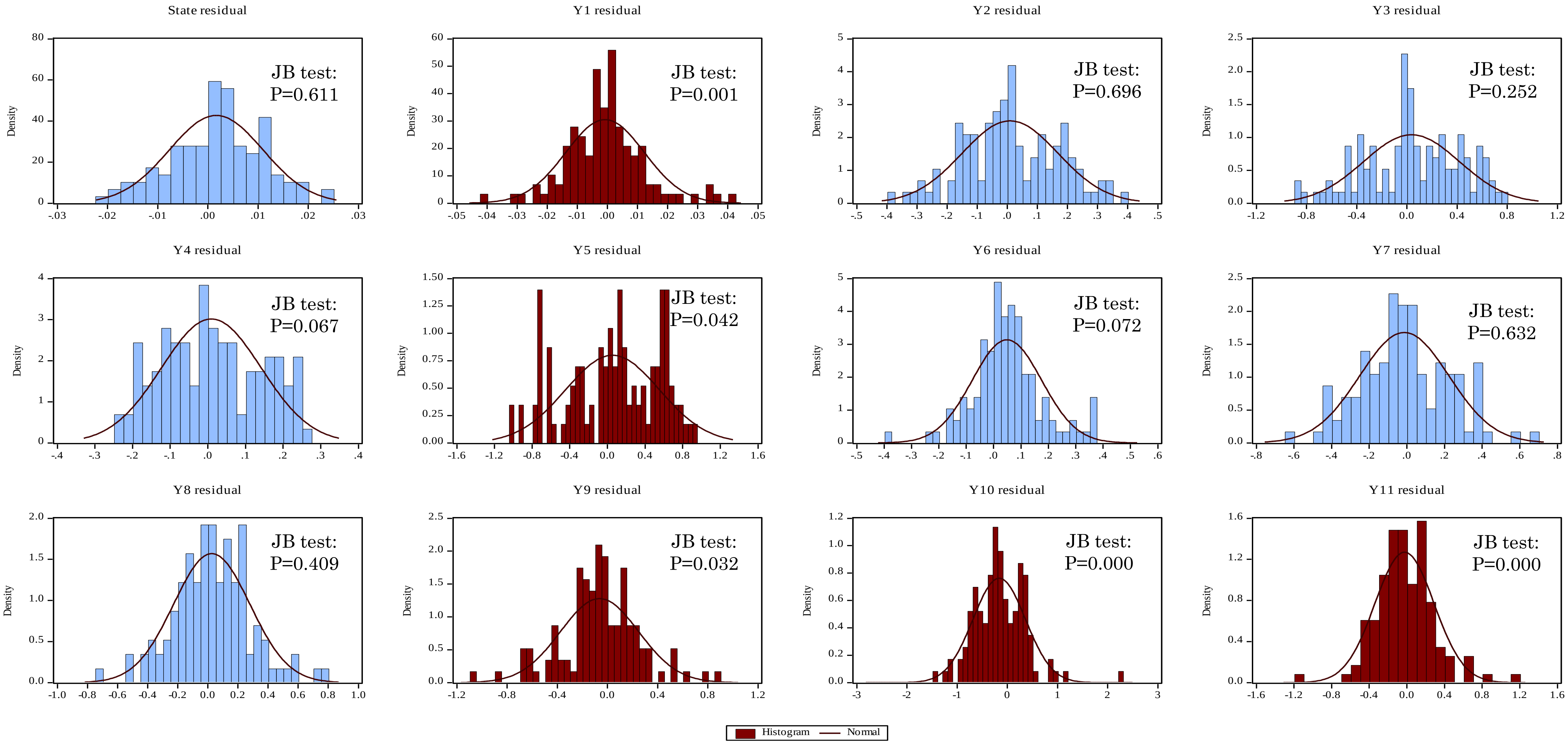
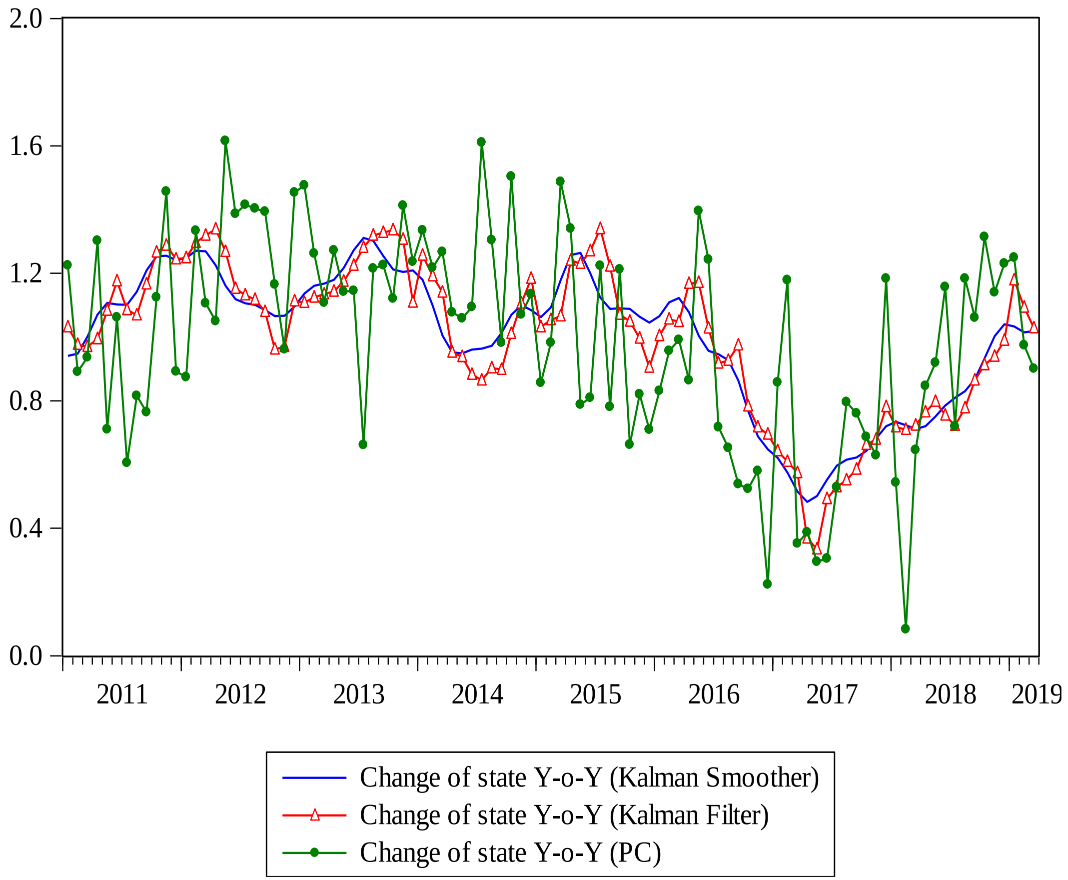
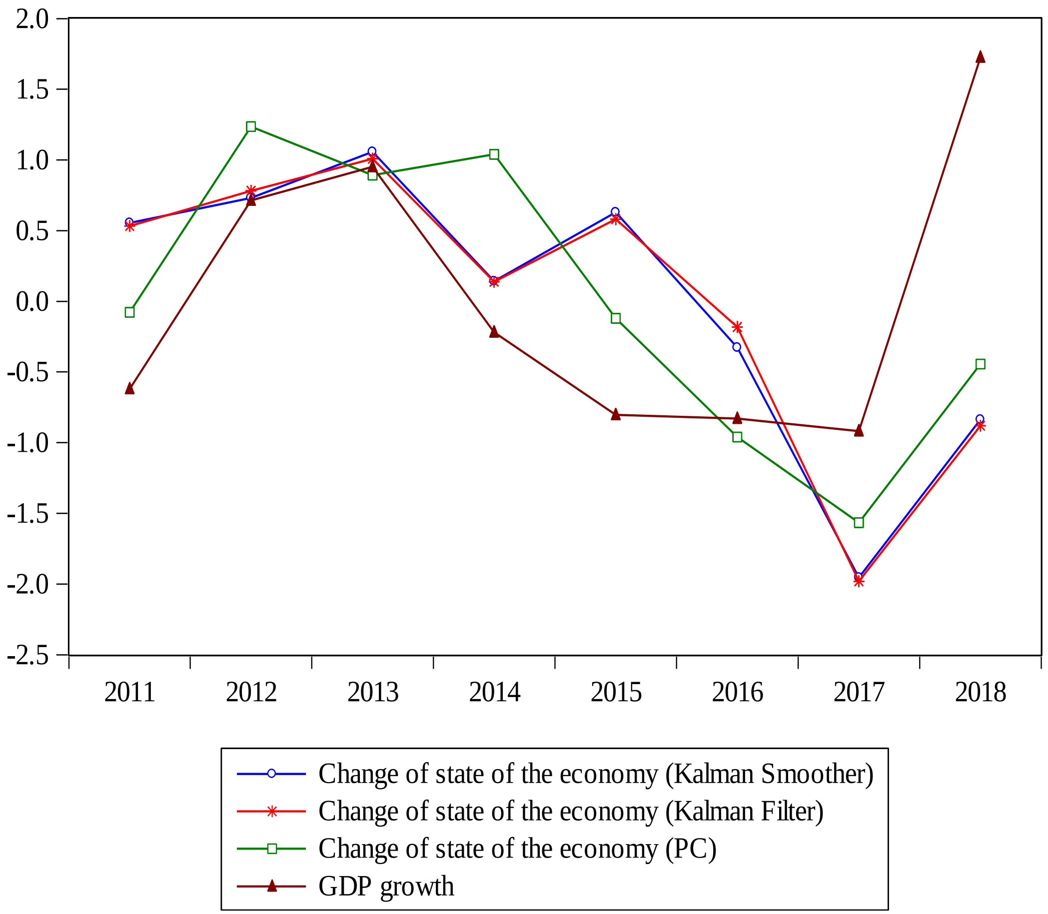


Figure 5 Comovement with GDP

Panel D



Panel B



2. Fiscal policy and state of the economy

- The ARDL (p, q) model:

$$\Delta Y_t = c + \delta t + \phi Y_{t-1} + \lambda X_{t-1} + \sum_{i=1}^{p-1} \omega_i \Delta Y_{t-i} + \sum_{i=1}^{q-1} \tau_i \Delta X_{t-i} + \gamma \Delta X_t + \alpha Z_t + \rho(S_t - S_{t,hp}) + \epsilon_t \quad (1)$$

- Different slope estimation

- $$\Delta Y_t = \begin{cases} a_0 + \delta t + \phi Y_{t-1} + \lambda X_{t-1} + \sum_{i=1}^{p-1} \omega_i \Delta Y_{t-i} + \sum_{i=1}^{q-1} \tau_i \Delta X_{t-i} + \gamma \Delta X_t + \alpha Z_t + \rho_1(S_t - S_{t,hp}) + \epsilon_t, & S_t > S_{t,hp} \\ b_0 + \delta t + \phi Y_{t-1} + \lambda X_{t-1} + \sum_{i=1}^{p-1} \omega_i \Delta Y_{t-i} + \sum_{i=1}^{q-1} \tau_i \Delta X_{t-i} + \gamma \Delta X_t + \alpha Z_t + \rho_2(S_t - S_{t,hp}) + \epsilon_t, & S_t \leq S_{t,hp} \end{cases} \quad (2)$$

- $$\Delta Y_t = b_0 + \mu I_t + \delta t + \phi Y_{t-1} + \lambda X_{t-1} + \sum_{i=1}^{p-1} \omega_i \Delta Y_{t-i} + \sum_{i=1}^{q-1} \tau_i \Delta X_{t-i} + \gamma \Delta X_t + \alpha Z_t + \rho_1 I_t (S_t - S_{t,hp})$$

$$+ \rho_2(1 - I_t)(S_t - S_{t,hp}) + \epsilon_t; \text{ where } I_t = \begin{cases} 1, & \text{for } S_t > S_{t,hp} \\ 0, & \text{for } S_t \leq S_{t,hp} \end{cases} \quad (3)$$

- Let $S_t^+ = I_t(S_t - S_{t,hp})$, and $S_t^- = (1 - I_t)(S_t - S_{t,hp})$

Table 4 The impact of the state of the economy on the fiscal stance (Conditional error correction form and bound test)

Dependent Var. ΔExpenditure	State (Kalman Smoother)		State (Kalman Filter)		State (principal component)	
	Coef.	Coef.	Coef.	Coef.	Coef.	Coef.
Expenditure (-1)	-2.033***	-2.490***	-2.049***	-2.553***	-2.342***	-2.671***
	(0.369)	(0.450)	(0.373)	(0.448)	(0.380)	(0.432)
State (-1)	-6.740***	-11.031***	-6.675***	-11.898***	-7.047***	-8.272***
	(1.364)	(3.252)	(1.405)	(2.865)	(1.281)	(2.077)
Revenue (-1)		0.137		0.106		0.151
		(0.251)		(0.240)		(0.250)
Money supply (-1)		-1.054		-1.312		-0.111
		(0.927)		(0.788)		(0.587)
ΔExpenditure (-1)	0.951***	1.265***	1.002***	1.352***	1.256***	1.440***
	(0.326)	(0.392)	(0.332)	(0.394)	(0.335)	(0.376)
ΔExpenditure (-2)	0.753**	1.013***	0.767**	1.063***	0.999***	1.165***
	(0.287)	(0.338)	(0.296)	(0.342)	(0.296)	(0.327)
ΔExpenditure (-3)	0.631**	0.813***	0.607**	0.830***	0.759***	0.888***
	(0.239)	(0.279)	(0.249)	(0.286)	(0.248)	(0.273)
ΔExpenditure (-4)	0.463**	0.595**	0.428**	0.591**	0.553***	0.639***
	(0.198)	(0.231)	(0.205)	(0.235)	(0.203)	(0.225)
ΔExpenditure (-5)	0.343**	0.440**	0.347**	0.444**	0.423***	0.468***
	(0.156)	(0.175)	(0.157)	(0.175)	(0.155)	(0.169)
ΔExpenditure (-6)	0.123	0.180	0.154	0.197*	0.181*	0.207*
	(0.110)	(0.118)	(0.109)	(0.115)	(0.108)	(0.115)
ΔState	-25.510***	-20.534**	-3.728	-8.617**	-6.037***	-7.175**
	(9.451)	(10.006)	(2.903)	(3.654)	(1.683)	(2.740)
ΔState (-1)	25.562**	11.562	-0.388	-0.107	0.818	0.908
	(13.227)	(15.018)	(2.534)	(2.688)	(0.847)	(0.970)
ΔState (-2)	-9.563	1.000	2.468	3.579	0.324	0.470
	(9.579)	(11.382)	(2.568)	(2.719)	(0.712)	(0.790)
ΔRevenue		0.108		0.110		0.108
		(0.092)		(0.091)		(0.092)
ΔRevenue (-1)		-0.005		0.003		0.005
		(0.157)		(0.151)		(0.155)
ΔRevenue (-2)		0.000		0.018		0.022
		(0.096)		(0.094)		(0.097)
Δ Money Supply		-2.810*		-2.944*		-2.225
		(1.633)		(1.603)		(1.606)
Δ Money Supply (-1)		-0.690		-0.655		-1.134
		(1.657)		(1.654)		(1.603)
Δ Money Supply (-2)		0.555		0.121		0.214
		(1.708)		(1.725)		(1.632)
ΔInterest rate		0.210		0.176		0.213
		(0.223)		(0.224)		(0.226)
Inflation		-0.089*		-0.097**		-0.109**
		(0.050)		(0.048)		(0.049)
Exchange rate		0.086		0.072		0.071
		(0.066)		(0.066)		(0.065)
Output gap	4.335	8.665*	5.784**	10.658***	7.114***	8.209**
	(3.318)	(4.665)	(2.537)	(3.432)	(2.040)	(3.170)
Election (dummy)	0.178	0.288*	0.217*	0.274**	0.104	0.156
	(0.122)	(0.141)	(0.117)	(0.123)	(0.112)	(0.115)
State (dummy)	0.079	0.054	-0.014	0.018	-0.085	-0.126
	(0.169)	(0.181)	(0.156)	(0.166)	(0.159)	(0.169)
Trend	0.263***	0.438***	0.263***	0.476***	0.281***	0.328***
	(0.051)	(0.133)	(0.052)	(0.116)	(0.049)	(0.085)
C	-14.583***	-24.396***	-14.627***	-26.372***	-15.536***	-18.020***
	(2.832)	(7.561)	(2.895)	(6.490)	(2.704)	(4.697)
R^2	0.573	0.622	0.559	0.624	0.583	0.638
Adjusted R^2	0.499	0.493	0.483	0.495	0.511	0.514
Prob(F-statistic)	0.000	0.000	0.000	0.000	0.000	0.000

Standard errors are in parentheses. Data are normalized with means 0 and standard deviation 1. Model selection is based on AIC value. Note. *; **; *** are significance at 10%, 5%, and 1%, respectively.

Table 5 The impact of the state of the economy on the fiscal stance (Different slope estimation)

Dependent Var. ΔExpenditure	State (Kalman Smoother)		State (Kalman Filter)		State (principal component)	
	Coef.	Coef.	Coef.	Coef.	Coef.	Coef.
Expenditure (-1)	-2.019***	-2.534***	-2.031***	-2.574***	-2.316***	-2.646***
	(0.370)	(0.452)	(0.374)	(0.450)	(0.384)	(0.437)
State (-1)	-6.693***	-11.357***	-6.609***	-11.997***	-6.952***	-8.304***
	(1.369)	(3.268)	(1.406)	(2.875)	(1.297)	(2.088)
Revenue (-1)		0.231		0.165		0.155
		(0.267)		(0.252)		(0.252)
Money supply (-1)		-1.155		-1.349*		-0.164
		(0.932)		(0.791)		(0.598)
ΔExpenditure (-1)	0.936***	1.302***	0.981***	1.370***	1.246***	1.434***
	(0.328)	(0.393)	(0.333)	(0.395)	(0.336)	(0.377)
ΔExpenditure (-2)	0.741**	1.047***	0.743**	1.073***	0.979***	1.149***
	(0.288)	(0.339)	(0.297)	(0.343)	(0.300)	(0.331)
ΔExpenditure (-3)	0.622**	0.848***	0.575**	0.834***	0.755***	0.887***
	(0.240)	(0.281)	(0.251)	(0.287)	(0.249)	(0.274)
ΔExpenditure (-4)	0.461**	0.630***	0.406*	0.594**	0.540***	0.627***
	(0.198)	(0.233)	(0.206)	(0.236)	(0.206)	(0.227)
ΔExpenditure (-5)	0.348**	0.472***	0.333**	0.447**	0.415***	0.459***
	(0.156)	(0.178)	(0.158)	(0.175)	(0.157)	(0.170)
ΔExpenditure (-6)	0.128	0.198	0.153	0.203*	0.171	0.195
	(0.110)	(0.119)	(0.109)	(0.116)	(0.110)	(0.118)
ΔState	-26.262***	-21.689**	-3.659	-8.743**	-5.910***	-7.228**
	(9.530)	(10.070)	(2.903)	(3.667)	(1.703)	(2.755)
ΔState (-1)	26.421**	12.201	-0.241	0.105	0.829	0.884
	(13.313)	(15.029)	(2.538)	(2.708)	(0.851)	(0.975)
ΔState (-2)	-10.751	-0.103	2.241	3.461	0.345	0.464
	(9.738)	(11.433)	(2.577)	(2.730)	(0.716)	(0.794)
ΔRevenue		0.136		0.117		0.114
		(0.096)		(0.092)		(0.093)
ΔRevenue (-1)		-0.039		-0.023		0.000
		(0.160)		(0.155)		(0.156)
ΔRevenue (-2)		-0.016		0.009		0.018
		(0.098)		(0.095)		(0.098)
Δ Money Supply		-2.953*		-2.964*		-2.338
		(1.639)		(1.608)		(1.628)
Δ Money Supply (-1)		-0.804		-0.782		-1.173
		(1.661)		(1.667)		(1.612)
Δ Money Supply (-2)		0.582		0.089		0.310
		(1.708)		(1.729)		(1.650)
Inflation		-0.085*		-0.092*		-0.105**
		(0.050)		(0.049)		(0.050)
ΔInterest rate		0.229		0.195		0.202
		(0.224)		(0.226)		(0.228)
Exchange rate		0.077		0.064		0.069
		(0.067)		(0.067)		(0.065)
Output gap (S^-)	7.507	13.654**	8.744**	13.170***	7.649***	8.933**
	(5.435)	(6.791)	(3.855)	(4.702)	(2.250)	(3.472)
Output gap (S^+)	2.758	6.371	3.963	9.166**	6.303**	7.603**
	(3.954)	(5.187)	(3.102)	(3.932)	(2.488)	(3.388)
Election (dummy)	-0.180	0.007	-0.241**	-0.011	-0.168	-0.124
	(0.123)	(0.187)	(0.119)	(0.170)	(0.113)	(0.170)
State (dummy)	0.040	-0.245*	-0.053	-0.294**	-0.084	-0.159
	(0.177)	(0.141)	(0.161)	(0.126)	(0.160)	(0.115)
Trend	0.262***	0.451***	0.261***	0.479***	0.277***	0.330***
	(0.051)	(0.133)	(0.052)	(0.117)	(0.049)	(0.086)
C	-14.394***	-24.936***	-14.429***	-26.500***	-15.314***	-18.097***
	(2.851)	(7.579)	(2.901)	(6.509)	(2.742)	(4.722)
R^2	0.575	0.627	0.564	0.627	0.585	0.639
Adjusted R^2	0.496	0.493	0.483	0.492	0.508	0.509
Prob(F-statistic)	0.000	0.000	0.000	0.000	0.000	0.000

Standard errors are in parentheses. Data are normalized with means 0 and standard deviation 1. Model selection is based on AIC value. Note. *; **; *** are significance at 10%, 5%, and 1%, respectively.

V. Discussion and limitation of the study

- First, it estimates the unobserved state in the linear context. For non-linear structure: extensions such as the Extended Kalman filter (EKF), Unscented Kalman Filter (UKF), and Particle filter (PF) can be used.
- Second, the study implements time-invariant parameters. (No structural break)
- Lastly, two problems regarding the ARDL model: 1^{st}, unique cointegration, and 2^{nd}, serial correlation and endogeneity problems.
- The ARDL could resolve the serial correlation and endogeneity issues by adding appropriate lags regressors, for example, ARDL (p, q) to ARDL (p, m), for $m \geq q$ (Pesaran & Shin, 1998).

VI. Conclusion

- The study constructs a coincident indicator, unobserved state of the economy, using the two-steps procedure proposed by Doz, Giannone, and Reichlin (2011).
- First, it estimates the parameters by the OLS and FGLS methods using the state variable generated by the PC. In the second step, it estimates the unobserved state via the DFM by substituting parameters with its consistent estimators.
- Second, it examines the relations between Cambodia's fiscal position and the state of the economy. All models show that the Cambodia fiscal position tends to be procyclical.

Thank You!!!