Question Booklet

Examination for Japanese University Admission
for International Students (Trial)

Mathematics (80min)

【Course 1・Course 2】

(Select either of these courses and answer their questions only.)

Ⅰ. Important Rules and Information

1. Do not open this question booklet until permission to start the examination has been given.
2. This question booklet has 24 pages.
3. You must mark your answers on the answer sheet with an HB pencil.
4. You may write notes in the margins of the question booklet.
5. You may not leave the room with this question booklet, even after the examination is over.
6. Write your name and examination registration number in space provided below, in the same way that they appear on your examination voucher.

Ⅱ. Answering Method

1. Each letter A, B, C, etc. in the questions represents a numeral (from 0 to 9) or the minus sign (−). Completely blacken your answer for each letter in the corresponding line of the answer sheet. Write square roots in their simplest form; for example, simplify $\sqrt{12}$ to $2\sqrt{3}$. When writing fractions, attach the minus sign to the numerator, and reduce the fraction to its lowest terms.

【Example】

If your answer to $\frac{\boxed{A}\sqrt{\boxed{B}}}{\boxed{CD}}$ is $\frac{-\sqrt{3}}{14}$, you would mark your answer as shown below.

【Answer sheet】

A	●	⓪	①	②	③	④	⑤	⑥	⑦	⑧	⑨
B	⊖	⓪	①	②	●	④	⑤	⑥	⑦	⑧	⑨
C	⊖	⓪	●	②	③	④	⑤	⑥	⑦	⑧	⑨
D	⊖	⓪	①	②	③	●	⑤	⑥	⑦	⑧	⑨

2. Be sure to carefully read the instructions on the answer sheets.

Examination registration number	
Name	

Mathematics: Course 1

Marking of Your Course Selection

You must indicate your selection of the course (Course 1 or Course 2) on the answer sheet. As shown in the example on the right, if you select Course 1, circle the label "Course 1" and completely blacken the oval under the label. If you do not properly blacken the appropriate oval, your answers may not be graded.

<Example>

Course	
Course 1	Course 2
●	○

I To fill the blanks **A**～**G** in the following statements (a)～(d), choose the most appropriate one from the choices ①～④ listed below each of them.

(a) The fraction $\dfrac{1+\sqrt{5}}{-2+\sqrt{5}}$ is equal to **A**.

① $-7-3\sqrt{5}$ ② $-7+3\sqrt{5}$ ③ $7-3\sqrt{5}$ ④ $7+3\sqrt{5}$

(b) Let x and y be real numbers. Among the following statements concerning x and y, an incorrect statement is **B**.

① If both of $x+y$ and xy are positive numbers, both of x and y are also positive numbers.

② If $x+y$ is an irrational number, at least one of x and y is also an irrational number.

③ If x and y are integers, and x^2+y^2 is an odd number, either x or y is also an odd number.

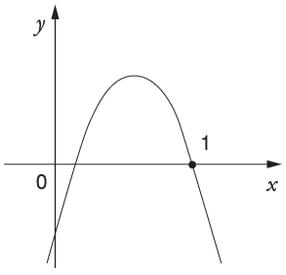
④ The inequality $|x|<|y|$ is a necessary and sufficient condition for the inequality $x^2<y^2$.

⑤ The inequality $x^2<y^2$ is a sufficient condition for the inequality $x<y$.

(c) Suppose the graph of the quadratic function

$$y = ax^2 + bx + c$$

is as shown in the diagram on the right. Then the value of a is C, the value of b is D, the value of $b^2 - 4ac$ is E and the value of $a + b + c$ is F.



① positive ② negative ③ 0 ④ otherwise

(d) When two dice are thrown simultaneously, the probability for which the pip 1 or 2 appears on at least one of the two dice is G.

① $\frac{7}{18}$ ② $\frac{4}{9}$ ③ $\frac{1}{2}$ ④ $\frac{5}{9}$ ⑤ $\frac{11}{18}$

II The symbol − (minus sign) or one of the digits: 0, 1, …, 9 corresponds to each of the letters A, B, …, R included in boxes of the following stasements (a)～(d). Choose an appropriate sign or digit for each of those letters.

(a) The roots of the cubic equation

$$x^3-3x^2-5x-1=0$$

are $x=\boxed{\text{AB}}$ and $x=\boxed{\text{C}}\pm\sqrt{\boxed{\text{D}}}$.

(b) If the parabola

$$y = ax^2 + bx + c$$

passes through the three points $(-1,\ 9)$, $(1,\ 1)$, $(2,\ 3)$, then

$a =$ E , $b =$ FG , $c =$ H .

(c) The polynomial

$$P=2x^2+2xy+y^2+4x+5$$

can be transformed to

$$P=(x+y)^2+\left(x+\boxed{\text{I}}\right)^2+\boxed{\text{J}}.$$

Therefore, it attains its minimum value when $x=\boxed{\text{KL}}$ and $y=\boxed{\text{M}}$, and the minimum value is $\boxed{\text{N}}$.

(d) The following equality holds:

$$\log_{10}225=\boxed{\text{OP}}\log_{10}2+\boxed{\text{Q}}\log_{10}3+\boxed{\text{R}}.$$

III The symbol $-$ (minus sign) or one of the digits: 0, 1, ..., 9 corresponds to each of the letters A, B, ..., O included in boxes of the following statements (a)～(c). Choose an appropriate sign or digit for each of those letters.

(a) Let C be the circle given by the equation

$$x^2+y^2+4x-6y-12=0.$$

i. The coordinates of the center of C is $(\boxed{\text{AB}}, \boxed{\text{C}})$ and the radius of C is $\boxed{\text{D}}$.

ii. If a tangential line of C passes the point $\mathrm{P}(6, -1)$, the distance between the point P and the point of tangency is $\sqrt{\boxed{\text{EF}}}$.

(b) A triangle PQR satisfies

$$PQ=5, \qquad PR=3, \qquad \angle P=60^\circ.$$

i. The area of $\triangle$ PQR is $\dfrac{\boxed{GH}\sqrt{\boxed{I}}}{4}$.

ii. The length of the edge QR is $\sqrt{\boxed{JK}}$.

iii. The radius of the circumcircle of $\triangle$ PQR is $\dfrac{\sqrt{\boxed{LM}}}{3}$.

(c) A regular triangle is given with edges of length 1. Consider the solid obtained by rotating this triangle with an edge as the axis of rotation. The volume of this solid is $\frac{\pi}{\boxed{\text{N}}}$ and the surface area of this solid is $\sqrt{\boxed{\text{O}}}\,\pi$.

IV The symbol − (minus sign) or one of the digits: 0, 1, ..., 9 corresponds to each of the letters A, B, ..., M included in boxes of the following statements (a)～(c). Choose an appropriate sign or digit for each of those letters.

(a) Denote by a_n the n-th term of the arithmetic progression

$$1,\ 8,\ 15,\ 22,\ \cdots,$$

i. It follows that $a_n = \boxed{\text{A}}\, n - \boxed{\text{B}}$.

ii. The sum of all terms from the first term to the 30-th term is $\boxed{\text{CDEF}}$.

(b) Let a be real number. The parallel displacement is applied to the parabola

$$C_1 : y = x^2$$

by 3 to the direction of the x-axis, and by a to the direction of y-axis. The resulting curve is denoted by C_2.

i. the x-coordinate of the point P of intersection of two curves C_1 and C_2 is $\dfrac{a+\boxed{\text{G}}}{\boxed{\text{H}}}$.

ii. If the tangential lines of C_1 and C_2 both at the point P intersect orthogonally, then $a = \pm\boxed{\text{I}}\sqrt{\boxed{\text{J}}}$.

(c) For a positive number a and a parabola

$$\text{C: } y=x^2$$

let S_1 be the area of the figure surrounded by C and two lines $y=0$ and $x=a$, and let S_2 be the area of the figure surrounded by C and two lines $y=a^2$ and $x=0$.

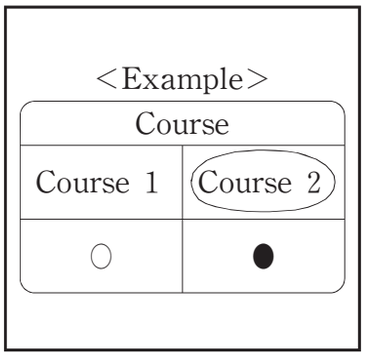
i. It holds that $S_1=\dfrac{a^{\boxed{K}}}{\boxed{L}}$.

ii. It follows that $S_1 : S_2=1 : \boxed{M}$.

Mathematics: Course 2

Marking of Your Course Selection

You must indicate your selection the course (Course 1 or Course 2) on the answer sheet. As shown in the example on the right, if you have selected Course 2, circle the label "Course 2" and completely blacken the oval under the label. If you do not properly blacken the appropriate oval, your answers may not be graded.



I To fill the blanks **A**～**G** in the following statements (a)～(e), choose the most appropriate one from several choices listed below each of them.

(a) The fraction $\dfrac{1+\sqrt{5}}{-2+\sqrt{5}}$ is equal to **A**.

① $-7-3\sqrt{5}$ ② $-7+3\sqrt{5}$ ③ $7-3\sqrt{5}$ ④ $7+3\sqrt{5}$

(b) Let x and y be real numbers. Among the following statements concerning x and y, an incorrect statements is **B**.

① If both of $x+y$ and xy are positive numbers, both of x and y are also positive numbers.

② If $x+y$ is an irrational number, at least one of x and y is also an irrational number.

③ If x and y are integers, and x^2+y^2 is an odd number, either x or y is also an odd number.

④ The inequality $|x|<|y|$ is a necessary and sufficient condition for the inequality $x^2<y^2$.

⑤ The inequality $x^2<y^2$ is a sufficient condition for the inequality $x<y$.

(c) The graph of the fractional function $y=\dfrac{x}{x-2}$ is obtained from the graph of $y=\dfrac{2}{x}$ by parallel displacement. This parallel displacement is $\boxed{\text{C}}$.

① by 2 in the positive direction of the x-axis

② by 2 in the negative direction of the x-axis

③ by 2 in the positive direction of the x-axis and by 1 in the positive direction of the y-axis

④ by 2 in the positive direction of the x-axis and by 1 in the negative direction of the y-axis

⑤ by 2 in the negative direction of the x-axis and by 1 in the positive direction of the y-axis

⑥ by 2 in the negative direction of the x-axis and by 1 in the negative direction of the y-axis

(d) For real numbers p and q, consider the matrix

$$M=\begin{pmatrix} p & q \\ 1 & -p \end{pmatrix}.$$

i. If $q=\boxed{\text{D}}$ then M does not have an inverse matrix.

ii. If $q=\boxed{\text{E}}$ then $M^2=I$. Here I denotes the unit matrix.

① $-p$ ② p^2 ③ $-p^2$ ④ p^2-1 ⑤ $1-p^2$

(e) A box contains 2 red balls and 3 white balls. A trial is to take one ball at random out of the box. Repeat this trial 4 times, without putting balls back to the box that have previously been taken out.

i. The probability for which 2 red balls are included in the set of 4 balls taken out is F.

ii. Subject to the condition that the ball taken out in the first trial is a white ball, the "conditional probability" for which 2 red balls are included in the set of 4 balls taken out is G.

① $\frac{1}{3}$ ② $\frac{2}{5}$ ③ $\frac{1}{2}$ ④ $\frac{3}{5}$ ⑤ $\frac{2}{3}$

II The symbol − (minus sign) or one of the digits: 0, 1, …, 9 corresponds to each of the letters A, B, …, M included in boxes of the following statements (a)〜(d). Choose an appropriate sign or digit for each of those letters.

(a) If a cubic equation of x with real coefficients

$$x^3-ax^2+bx-4=0$$

has a complex root $1+i$, then $a=$ A and $b=$ B.

(b) Let C be the circle given by the equation

$$x^2+y^2+4x-6y-12=0.$$

i. The coordinates of the center of C is $\left(\boxed{\text{CD}}, \boxed{\text{E}}\right)$ and the radius of C is $\boxed{\text{F}}$.

ii. Draw a tangential line of C which passes the point P(6, -1). The distance between the point P and the point of tangency is $\sqrt{\boxed{\text{GH}}}$.

(c) The solutions of the equation

$$(\log_2 64x)\left(\log_2 \frac{16}{x}\right)=24$$

are $x=$ I and $x=\dfrac{1}{\text{J}}$.

(d) For the function

$$y=4\sin x+2\cos 2x-2$$

the maximum value is $\boxed{\text{K}}$ and the minimum value is $\boxed{\text{LM}}$.

III The symbol − (minus sign) or one of the digits: 0, 1, …, 9 corresponds to each of the letters A, B, …, N included in boxes of the following statements (a)～(c). Choose an appropriate sign or digit for each of those letters.

(a) Consider a progression

$$1,\ \underbrace{2,\ 2,}_{\text{2 terms}}\ \underbrace{3,\ 3,\ 3,}_{\text{3 terms}}\ \cdots,\ \underbrace{n,\ \cdots,\ n,}_{\text{n terms}}\ \cdots.$$

i. If N is the 70-th term of this progression, then N=[AB].

ii. There are [C] terms that is equal to N among the first 70 terms.

iii. The sum of the first 70 terms is [DEF].

(b) If the lengths of two vectors $\vec{a}$, $\vec{b}$ are 2, 3 respectively, and the angle formed by them is 60°, then the inner product of $\vec{a}$ and $\vec{b}$ is $\boxed{\text{G}}$ and the length of the vector $\vec{a}+2\vec{b}$ is $\boxed{\text{H}}\sqrt{\boxed{\text{IJ}}}$.

(c) Consider the cubic function

$$f(x)=x^3+ax^2+bx+4$$

with real numbers a, b as its coefficients. If the derivative $f'(x)$ satisfies

$$f'(x)=3x^2+2x\int_0^2 f(x)dx+4,$$

then

$$a=\boxed{\text{KLM}}, \qquad b=\boxed{\text{N}}.$$

IV The symbol $-$ (minus sign) or one of the digits: 0, 1, …, 9 corresponds to each of the letters A, B, …, K included in boxes of the following statements (a), (b). Choose an appropriate sign or digit for each of those letters.

(a) For the function

$$y=\frac{x^2+3}{x+1}$$

its derivative is

$$y'=\frac{x^2+\boxed{\text{A}}x-\boxed{\text{B}}}{(x+1)^2}.$$

Therefore the values x for which $y'=0$ holds are $x=\boxed{\text{CD}}$ and $x=\boxed{\text{E}}$.

Accordingly, the minimal value of y is $\boxed{\text{F}}$ and the maximal value of y is $\boxed{\text{GH}}$.

(b) For the function

$$y=x+2\sin x$$

its differential coefficient at $x=0$ is $\boxed{\text{I}}$. Within the range of $0\leqq x\leqq\pi$, $y'=0$ is satisfied when $x=\dfrac{\boxed{\text{J}}}{\boxed{\text{K}}}\pi$.